

We Talk About It, But Do They Get It?

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Abstract

This article reports on the last of a series of iterative research studies involving students with learning disabilities in reform mathematics classrooms at the intermediate grade levels. This study reports the findings from a larger, yearlong case study, focusing on ways to include students with learning disabilities and other students who are at risk for special education services in classwide discussions of problem solving. The data reported in this article detail the changes in teacher and student discourse over a nine week period in one classroom. Sources of data for this study included videotapes, audiotapes, and informal interviews with the teacher, paraprofessional, and students. A quantitative analysis of the results indicates clear patterns of change in teacher and student discourse. Nonetheless, intentional efforts to include target students in the whole class discussions yielded instructional dilemmas that are under-described in the mathematics reform literature. Findings from this study have implications for special educators interested in mathematical problem-solving, as well as math reformers who value the role of classroom discourse in daily instruction.

Introduction

Policy documents such as the National Council of Teachers of Mathematics' *Curriculum and Evaluation Standards* (NCTM, 1989) and *Principles and Standards for School Mathematics* (NCTM, 2000) as well as any number of research studies conducted over the last decade argue for the importance of new discourse practices in reform mathematics classrooms (e.g., Ball, 1993; Cobb, 1999; Lampert, 1997; Lampert & Blunk, 1998; Lampert, Rittenhouse, & Crumbaugh, 1996). Carefully orchestrated, classroom discourse can model mathematical reasoning and problem solving for students. It can also assist students' understanding of key ideas and enhance their disposition toward mathematics (National Research Council, 2001).

Reform-based discourse practices generally present a model of teacher-student communication that is markedly unlike the Initiation-Response-Evaluation or "IRE" method that appears in so many American classrooms. The problems with the IRE method as an exclusive form of classroom discourse are well-documented (Cazden, 1988; Mehan, 1979). For example, teachers ask innumerable questions to which the answers are already pre-established. Students are drawn into a pattern of guessing the answer that the teacher already had in mind when he or she asked the question. Finally, students' answers to questions tend to be brief in nature, including simple "yes" or "no" responses.

In discourse driven, reform mathematics classrooms, students spontaneously debate concepts among themselves while the teacher listens or moderates the discussion. Teacher talk often involves probing students in a careful, but persistent manner in the effort to develop greater conceptual understanding. It is not uncommon for teachers to spend a considerable amount of time conducting a dialogue with one or two students and,

at times, restate or “revoice” a student’s claims into more formal mathematical vocabulary as one of many subtle ways of refining mathematical thinking (O'Connor & Michaels, 1996).

Accounts of collaborative efforts in reform mathematics attest to how labor intensive this process is for classroom teachers (Ball & Rundquist, 1993; Cobb, Wood, & Yackel, 1993; Heaton & Lampert, 1993). A successful move away from the IRE model toward more contemporary, constructivist practices depends, in large measure, upon high levels of pedagogical and content knowledge. As Williams and Baxter (1996) also note, teachers need to provide students with social scaffolding that supports productive mathematical conversations in the classroom. For example, teachers need to model thoughtful questioning, and they should explicitly encourage students to verbalize their ideas to better understand their own thinking.

A further and largely undocumented challenge for teachers is how to include all students in classroom discussions. A teacher’s persistent effort to develop mathematical concepts in a discussion not only takes time, but it may involve only a minority of students in the class. Our past research (Baxter, Woodward, & Olson, 2001) indicates that academically low achieving students typically remain passive in whole class discussions, which tend to be dominated by highly verbal, capable students. When low achieving students do participate, their answers tend to be simple, IRE-like responses or “I don’t know.” Ball (1993) alludes to the problem of getting academically low achieving students to successfully contribute to discussions as one of the dilemmas of mathematics instruction. At times, she notes that it is difficult to discern, “what some students know or believe -- either because they cannot put into words what they are

thinking or because *I* cannot track what they are saying” (p. 387). Similar findings stem from analyses of Lampert’s fifth grade classrooms. Chard’s (1999) analysis of Lampert’s classroom videotapes indicates that the contributions of academically low achieving students were typically meager and, at times, incomprehensible.

Purpose of the Study

The purpose of this study is to examine the nature of discourse practices that arise around problem solving instruction in one classroom when the teacher intentionally works to include a wide range of students during math discussions. The results reported here are part of a larger case study of four fourth grade classes, and the larger study includes standardized and criterion assessments, an attitude survey, interviews with teachers and aides, and detailed observations of discourse practices (Woodward & Baxter, 2001). This latter component of the larger case study – the detailed observations of discourse practices in the classroom – involves multiple methods of data collection, which include field notes, videotape recording, audiotaping, and analyses of the transcribed video and audiotapes.

This article examines three features of classroom discourse from one of the participating fourth grade classrooms: 1) how the discourse evolved over time from teacher-directed to student-centered discourse 2) the differences in discourse between capable students and academically low achieving students and 3) how the verbal interactions between target students, their peers, and the teacher impacted the way the teacher mediated the discussion. Through an analysis that concentrates on the large group discussions, we have been able to describe one type of non-narrative discourse

where individual phrases and statements are bound within a hierarchical structure of explanation and justification (see Georgakopoulou & Goutsos, 1997).

Method

Participants

Teacher. The teacher described in this study was one of four veteran, fourth grade teachers who participated in the larger case study. “Roberta Nelson” had over 20 years of elementary school teaching experience, and she had taught reform mathematics curricula for the last nine years. Her beliefs about mathematics instruction, as measured by Mathematics Beliefs Scale (Fennema, Carpenter, & Loef, 1990) were highly constructivist. The nature of her practices, at least as they reflect constructivist beliefs, are documented throughout the observations conducted for this study, and they will be described in the results section of this article.

Students. Twenty-eight students in Nelson’s class participated in this study. The average performance of these 28 students on the problem solving subtest of the Iowa Test of Basic Skills (ITBS) (1996) was at the 51st percentile. Shawna and Leslie, two of the three target students in this study, scored below the 25th percentile on this subtest (22nd and 19th percentiles respectively). These students were considered at risk for special education services in mathematics at the school. Jacob, the third student, had an IEP in math, and he was receiving his special education services through the aide in Nelson’s classroom. His score on the problem solving subtest of the ITBS was at the 6th percentile.

Materials

Daily curriculum. The elementary school in this study had been implementing *Everyday Mathematics* throughout the K-6 grade levels. The school adopted the program in the early 1990s.

Everyday Mathematics reflects over six years of development efforts by mathematics educators at the University of Chicago School Mathematics Project (UCSMP). This program de-emphasizes rote computational skills, and it differs from many traditional elementary math curricula in the way concepts are introduced and then reintroduced within and across grade levels. Students learn core mathematics concepts through a “concentric ring” approach. That is, major concepts are presented initially and then reappear later in the year and in the next grade level, where they are addressed in greater depth. For example, fractions are introduced in the first grade informally through manipulatives. Then, over the subsequent grades, there are more substantive investigations of fractions, and the instructional activities become increasingly more formal and symbolic.

The *Everyday Mathematics* program has a variety of problem solving activities. Some of the activities are brief, workbook exercises that are not unlike the kinds of problems found in traditional math programs. Other problems require a greater degree of application and draw on the students’ everyday world or from life science, geography, or other school subjects. Still other problem solving exercises require group work and may take a major portion of the period or even several days.

However, there is very little in *Everyday Mathematics* that involves the written communication required for the kind of problem solving found on the Washington

Assessment of Student Learning (WASL), which is administered at the fourth, seventh, and tenth grades. The WASL includes “extended response” items, and they require students to work problems and explain how they derived their answers. This format is missing in *Everyday Mathematics*, and it is one of the reasons why the fourth grade teachers were willing to devote one day per week on performance assessments throughout the seven months of this study.

Performance assessment exercises in Nelson’s classroom. Nelson and the three other teachers in the larger case study have worked with the researchers on this project for five years on a variety of mathematics intervention studies. They have worked collaboratively over that time, and in the previous two years, they met on a biweekly basis to discuss strategies for helping academically low achieving students and mainstreamed students with learning disabilities in their classrooms. The methods for teaching the performance assessments in this study evolved from the previous two years’ collaboration.

Nelson, as well as the three other teachers in the larger case study, supplemented her *Everyday Mathematics* lessons with performance assessment exercises. Researchers developed the majority of these weekly assessments, and on other occasions, selected problems from a variety of problem solving materials. Frequently, the content of the weekly problem related to the unit of instruction at the fourth grade level in *Everyday Mathematics*.

The problems were typically written with a common format so that students would have to explain how they derived their answers. The statement of the problem appeared at the top of the paper. The question and directions to describe the answer

using words, numbers, or pictures appeared below. Finally, a large blank box occupied the majority of the paper. It should also be noted that there was no attempt to calibrate the difficulty of the performance assessment items in terms of problem solving difficulty other than the fact that the teachers reviewed each problem beforehand for its difficulty and relevance to the *Everyday Mathematics*.

Procedures

Shawna, Leslie, and Jacob, the three target students in Nelson's classroom, received additional instructional assistance in the form of ad hoc tutoring. They also participated in the weekly classwide performance assessments, where paraprofessionals assisted them as they worked in small groups and, to a smaller extent, in the class discussions.

Ad hoc tutoring. Throughout the week, the target students received ad hoc tutoring from a paraprofessional aide. Tutoring varied depending upon the needs of the student, and generally, the aide worked with no more than the three students at once. Two teachers shared the services of the aide at one time, so tutoring was not an everyday event in Nelson's classroom.

Nelson determined the content of the tutoring based on an analysis of the curriculum unit (e.g., fractions, measurement, geometry) and on ongoing student performance in the classroom. The aide re-taught or reviewed fundamental concepts from the unit (e.g., the relationship of a fraction to a decimal; geometric properties such as angles, parallel lines, rays) and assisted students with workbook exercises. They also spent time teaching math facts.

The tutoring occurred in different contexts. On some occasions, tutors worked with target students in the classroom during the math lesson. On other occasions, they worked with students before or after the math lesson (e.g., one half hour before school, during the time other students were in band practice).

Performance assessments. All of the students in Nelson's class practiced performance assessments once a week. The teacher began each lesson by introducing the problem and stimulating background knowledge or adding a "real life" context. The discourse structure for this segment of the lesson generally followed the IRE format described by Mehan (1979) and others. Nelson's main intent was to make sure that students understood the problem and that they knew what to do during the next segment of the lesson. This portion of the lesson usually lasted 10 minutes.

The instructional aide assisted Nelson in this initial phase of the lesson by making sure that the target students were listening to the teacher's presentation. She also assisted Nelson by helping the target students transition to small group work (e.g., making sure that they knew how to get started, that they had appropriate materials).

In the second segment of the lesson, students worked in small groups for approximately 20 to 25 minutes. Groupings were frequently a function of the way that the desks were clustered in the classrooms. However, Nelson, like the other teachers in the larger case study, allowed students to choose a partner or small group to work on the problem. Nelson's target students were distributed throughout the five or six classroom groups on most of the weekly performance assessment lessons.

Nelson moved from one group of students to the next to make sure that there was an appropriate level of task engagement. She also asked questions, scaffolded understanding, and helped students summarize their progress on the problem.

During the small group interactions, the paraprofessional typically worked with the target students. The nature of assistance varied. Primarily, her job was to: 1) help them understand the problem and find a strategy for solving it, 2) listen carefully to other students or to make contributions, 3) make sure that the target student(s) actively participated in the problem solving (e.g., offering a suggestion for how to solve a step in the problem), and 4) rehearse their understanding of the group's strategy and solution before the next phase of instruction (i.e., the reporting out phase). The amount of aide-target student interaction also varied, from brief contacts to assistance that could last as long as five minutes. Overall, the aide helped support target students so that they did not adopt purely passive roles. She also assisted these students by modulating the difficulty or "cognitive load" of the problem.

By the end of the small group work, students demonstrated their understanding of the problem to the rest of the class by using a variety of media. At times, students would simply answer the problem on the worksheet using the blank space once the group had achieved some kind of consensus. Even though each student completed the answer individually, the response generally reflected group thinking. On other occasions, Nelson would have the group compose an explanation on a whiteboard or overhead transparency.

The final segment of the problem solving lesson involved a reporting out phase. The purpose of this segment was to give different groups of students the opportunity to share their answer and solution methods in front of the class. A secondary purpose of the

reporting out phase was to have the class see multiple solutions to the problem. On a few occasions, particularly in the beginning stages of the weekly lessons, Nelson even created “prototypic” answers before the lesson that she shared with students via the overhead projector at the end of the lesson.

Aide responsibilities during the reporting out phase were to monitor target students to make sure that they attended to what others said. At times, the aide also prepared a target student for reporting on their groups’ strategy and solution to the problem.

Data Collection and Analysis

The data sources that are relevant to this portion of the larger case study are: 1) video recordings of weekly problem solving, 2) audiotapes of small group interactions involving selected target students, 3) field notes of classroom interactions, and 4) informal interviews with teachers and aides.

Researchers placed a video recorder in the corner of the room in order to capture whole class interactions. The senior researcher on the project also moved about the room during small group interactions to record target students using an audio recorder attached to a clipboard. The researcher also took field notes during the lesson and added to these notes immediately afterwards. In order to clarify the nature of an interaction (e.g., a conversation between a target student and an aide or teacher), the researcher asked the adult about the substance of the interaction at a suitable point following the interaction. Finally, the researcher collected and photocopied student work samples where possible. Large presentations such as those written on sheets of butcher paper were not collected. However, they were recorded on the videotape and transcribed for later analysis.

The researcher interviewed the aide and teachers following the lesson or at other times during the school day. The researcher also met with the four teachers on a regular basis, and this was an opportunity to collect their impressions of the lessons and how they felt that they were meeting the needs of the academically low achieving students. The researcher and the teachers discussed how best to use the instructional aide during the small group and classroom discussion portions of the lesson. The group also discussed the progress of the target students in problem solving as well as their overall performance in mathematics.

Researchers transcribed all of the videotapes of the weekly problem solving for Nelson's classroom. Once the videotapes were transcribed, the goal of the analysis was to identify systematic patterns in the teachers' and students' statements. Researchers developed a coding scheme for this analysis that was grounded in past research in mathematical discourse (e.g., Lampert & Blunk, 1998; O'Connor & Michaels, 1996; Williams & Baxter, 1996), classical descriptions of IRE discourse patterns (Cazden, 1988; Mehan, 1979), and theoretical discussions of non-narrative forms of discourse (Georgakopoulou & Goutsos, 1997).

Researchers also reviewed field notes, classroom audio recordings of small group interactions, and student artifacts such as individual responses to the weekly problem solving exercises from Nelson's class. These additional sources of data enriched the analysis of classroom discourse as it evolved over the course of the eight months of the study.

Results

What follows are analyses of discourse patterns during the initial stages of the problem solving (i.e., the third week of the intervention) and again, six weeks later when the discourse had become more student centered (i.e., the ninth week of the intervention). We concentrate on the reporting out phase of each lesson because this was the primary context for verbal interactions between the teacher and students. Data are reported in terms of whole class patterns of discourse and more specific issues related to target student discourse.

Week 3 of Classroom Discourse

Whole class patterns. Figure 1 below shows the problem that the students were asked to solve on November 30, which was the third week of the intervention. The structure of the class followed the pattern that was common to the weekly problem solving lessons: orientation to the problem, small group interactions, and a reporting out phase that lasted just over 16 minutes. During the reporting out phase, Nelson called on three different groups of students to describe the strategies they used to solve the problem. Each group of students used the whiteboards that they had been given to record their work. This was an important prompt that students used to explain their strategies and respond to questions from the teacher and the other students in the class. In this lesson, each group showed a different strategy for solving the problem.

Insert Figure 1 about here

Our analysis of the third week lesson suggested that the teacher highly controlled the classroom talk. Nelson determined the discussion topics and the order in which students would speak. Nelson would call a group of students to the front of the class with their white board to present their solution to the problem. Once one or two students from the group described the strategy, she would ask questions to clarify their explanation, and then ask students in the class if they had any questions and if the group's strategy was clear. Nelson was responsible for almost 60 percent of the utterances. Figure 2 below exemplifies the kind of dialogue between the teacher and students.

Insert Figure 2 about here

Throughout the reporting out phase, there were neither student-to-student questions nor comments. Almost half of the teacher's comments (46 percent) were coded as either managing behavior and the flow of events or as clarifying student statements. The students tended to react to the teacher's questions, rather than initiate conversations with their own questions. Students' comments fell primarily into two categories: claims with no support (29 percent) and reporting on how they derived their calculations (47 percent).

To a large extent, Nelson's discourse patterns resemble the IRE model that is the cornerstone of American classrooms (Cazden, 1988; Mehan, 1979). Interviews with Nelson and subsequent, weekly analyses of her classroom discourse suggest that she is shaping the talk so that it can be more student-centered. For example, 23 percent of her statements related to social scaffolding. These were remarks intended to help students communicate

their thinking (e.g., “Try to remember we're trying to be more than problem solvers. We're trying to communicate to others how we solve problems. Can you see clearly what their strategy was and what their solution is?”). Also, nearly one-fifth of her statements (19 percent) prompted mathematical thinking (e.g., “From looking at their white board, do you understand their strategy? This is the time to look at their strategy and understand what they're doing. Teri, do you understand what they're doing?”). These statements were intended to get students to reflect on and explain their thinking as well as have them understand others' mathematical ways of thinking.

Target student interactions. Nelson provided the opportunity for all three of the target students to participate in the presentations and class discussion. Their involvement was highly variable. Nelson's only interaction with Shawna, one of the academically at-risk students, was to request that she stop writing and listen to the first group's explanation. Leslie, who tended to ask the greatest number of questions in the problem solving lessons throughout the seven month intervention, stood quietly with her group as two other students explained the group's strategy. She also volunteered to comment on one of the group's work, as is shown in Figure 3 below.

Insert Figure 3 about here

Jacob, the mainstreamed student with learning disabilities in math, had the greatest opportunity to talk and explain his strategy in this lesson. Figure 3 shows his effort to explain his group's strategy using a whiteboard. The dialogue between Nelson and Jacob reveals answers that focus on computational methods and do not elaborate on

the strategy used to derive the answer. It should be noted that Jacob had assistance from the aide during the small group portion of the class, and he is presenting a highly abbreviated account of the group's effort to solve the problem.

What is apparent in the Week 3 observations is that Nelson is trying to include the target students in the classroom discussions. She makes sure that they present in front of the class, that they answer questions, and that they are called upon when they volunteer. Even though Jacob's contributions are less clear than his peers, like the rest of the presenters, he still focuses on computational methods rather than strategies.

Week 9 of Classroom Discourse

Whole class patterns. Over the course of the six weeks following the lesson described above, Nelson's problem solving discussions became more student-centered. The lesson on January 26, provided a clear example of the kind of student-centered discussion that persisted for the majority of the lessons in the remainder of the study. Figure 4 shows the problem used for the day's lesson. This problem contained a level of calculation that was comparable to those required for the problem used during the third week's lesson. The overall organization of the lesson – orientation, small group work, and reporting out phase – was also the same as the third week's lesson. Three groups of students used overhead transparencies in this lesson rather than whiteboards to communicate their strategies and solutions to the problem.

Insert Figure 4 about here

Nelson continued to talk the majority of the time (56 percent), but the nature of her talk shifted considerably. As Figure 5 below indicates, Nelson’s social scaffolding diminished sharply, and her prompting of mathematical reflections increased considerably.

Insert Figure 5 about here

More evident were the categorical changes in the student talk. Students were far more prone to support their statements (or claims), rising from no occasions during the third week lesson to 13 percent of the time in week nine. Students were much more adept in this lesson at using mathematical vocabulary (e.g., “a multiple of five”, “product”, “a combination of guess and check and draw a picture”, “the way they communicated their solution”). They were also more likely to describe the strategies they used to solve the problem and to focus far less on reporting their calculation methods. These patterns are displayed in Figure 6 and 7 when Nelson asks the students, “And what about the way they communicated their strategies? Can you follow their strategies? Can you understand? How about you Leslie?” The reporting out phase was four times longer than this phase of the lesson in November, largely because of the increased length of student statements.

Insert Figure 6 about here

Finally, the student-centered nature of the discussion is apparent in Figure 7. As the students in the front of the class finish their presentation, Nelson guides students in the class to ask critical questions. Leslie, one of the academically low achieving target students in the class, initiates the questioning. Almost immediately, there is a rapid exchange between other students in the class, and Nelson acts to mediate and focus the questioning. By asking how the students had arrived at their answer, the students in the audience helped those presenting to focus and clarify the group's thinking. This kind of student-to-student exchange was missing in the initial phases of the weekly problem solving lessons.

Insert Figure 7 about here

It is also apparent in this lesson that Nelson has effectively created conditions where students feel comfortable commenting on each other's work. One student's comment later in the lesson is indicative of the kind of commentary and criticism found in the math discourse literature. When asked about her thoughts on the strategy that one group used, she stated, "I think they should sort of explain their ideas if this was going to go to like a grading place. It [the grading place] probably wouldn't understand that this was a list and they thought that they were going to add it then they wouldn't count it because they didn't put all the signs showing they were going to add it." Other students freely stated that they were confused by parts of the display on the overhead and offered suggestions for how the display could be improved so that the communication would be enhanced.

Target student interactions. Two of the target students (Leslie and Jacob) were given opportunities to participate in this discussion, either to raise questions while they were listening to other group presentations or to be part of a group that presented at the front of the room. In fact, their participation in this lesson is strikingly similar to Week 3 (e.g., Jacob speaks in front of the class with his group, Leslie asks questions of another group while she is at her desk). Shawna did not have a significant role in this lesson because a special education aide worked with her for part of the period on reading activities.

In this lesson, Jacob was the most involved of the three target students. The paraprofessional aide worked with Jacob for several minutes during the small group interaction phase, and she monitored how he attended to the subsequent class discussion in the reporting out phase. Figure 8 shows a reproduction of the overhead that Jacob's group used during their presentation to show their strategy and solution to the problem. Jacob's attempt to report the strategy that his group used is presented in Figure 9.

Insert Figures 8 and 9 about here

Jacob's statements focus on the pattern of numbers at the top and bottom of the display. Not only do these observations attend to marginally relevant features of the group's solution, but he repeats versions of the same comment on two additional occasions. His remarks clearly contrast with the verbal abilities of average achieving students like Sharon (see Figure 9). Sharon is able to restate the problem as well as concisely state how the group achieved the solution (i.e., by making an organized list). Finally, even though the teacher listens to Jacob, she moves quickly to capture Sharon's explanation for the class. Nelson wants to emphasize the strategy of making an organized list.

Informal interviews conducted throughout the intervention with Nelson and the other fourth grade teachers who participated in this research help clarify the problem that can occur when academically low achieving students like Jacob persist in giving confusing or marginally relevant explanations. During one interview (2/23/2001), Nelson remarked that it was difficult to sustain a high quality discussion when "students who are

struggling” have a hard time explaining their thinking or they interrupt the flow of the discussion with comments that sidetrack important ideas. Nelson, like the other three teachers who participated in this research, often found it difficult to stop and spend the time needed to understand everything that students like Jacob were saying.

What Nelson described as even more problematic was the “tension” between calling on a wide range of students in the class and still focusing the discussion on the best strategies and explanations. She noted that calling on a range of students served the function of making sure that students were paying attention and understood what was being said. For Nelson, focusing the discussions on the best strategies and explanations provided the best opportunity for everyone to see and understand different ways to solve the day’s problem and how to communicate it.

It should also be mentioned that not all of the target student’s comments were marginal nor was their participation limited to the examples described above. Leslie worked diligently in her small group, and as indicated in Figure 7, she asked a highly relevant question about another group’s solution method. She stood at the front of the class with other members of her group, but she did not present nor did she answer teacher or classmates’ questions. Nelson called on her on another occasion, but she did not know the answer to the question. While she raised her hand on two other occasions during the reporting out phase, Nelson did not call on her. As mentioned earlier, Shawna did not have a significant opportunity to participate in this problem solving lesson because she was receiving remedial assistance in reading in the corner of the room from a special education paraprofessional.

Discussion

The reform mathematics literature provides many different pictures of classroom discourse. Some accounts (e.g., Lampert & Blunk, 1998; O'Connor & Michaels, 1996) focus on subtle techniques that teachers use to scaffold student understanding and sustain in-depth discussions of mathematical topics. Others (e.g., Ball, 1993) detail key dilemmas that arise in discourse driven classrooms. The analysis presented in this study concentrates on the shift from teacher-centered to student-centered classrooms *with a consistent attempt* to include academically low achieving students as part of classroom discussions.

Detailed analyses of lessons over the course of the seven month study indicate that Nelson was able to achieve a more student-centered discussion for the weekly problems, at least as far as students were able to make supported claims as part of their explanations and freely exchange ideas and critiques of other students' strategies and solutions. The initial weeks may have appeared to be "IRE-like" in structure, but informal interviews as well as an analysis of her classroom discourse over time indicate that Nelson was carefully developing a new form of talk around a complex task (i.e., working on one problem for an entire class period). As the data indicate, she devoted a significant amount of time to social scaffolding in the early lessons.

One could argue that much of Nelson's success was facilitated by her considerable history in teaching reform mathematics programs as well as her constructivist views of learning. Another factor that may have facilitated this shift was the discrete format of the weekly lesson. By the ninth week, the class appeared comfortable with the three phases of instruction: orientation, small group work, and reporting out. It had become a predictable routine for the teacher and the students.

A focus on the academically low achieving students in classroom discussions rarely appears in the mathematics discourse literature, and the level of target student participation in Nelson's classroom is far greater than our earlier, naturalistic accounts of reform-based instruction (e.g., Baxter et al., 2001). Using paraprofessionals to assist these students during their small group work (e.g., helping them be involved in the small group discussion, assisting them with the problem solving, having the student rehearse what they would say in a presentation) undoubtedly improved the level of participation during the reporting out phase. Nelson's intentional effort to call on these students during the last phase was also crucial. Nonetheless, at least two dilemmas arise from the observations of Nelson's classroom.

First, as Nelson mentioned in an informal interview, there is a tension between calling on a wide range of students and sponsoring a fruitful, mathematically in-depth discussion. In most of Nelson's lessons, it is apparent that the more verbal and more capable students make the most thought-provoking or analytic contributions to the discussion. This is only natural, if not tautological. Increasingly, these students are the most likely to use appropriate mathematical vocabulary, and their commentary is often vital to Nelson's effort to "unpack" a novel strategy or make the communication of a solution clear to the other students in the class.

As Nelson intimated in the interview, moving back and forth from a core group of verbal students to academically low achieving students like Shawna, Leslie, and Jacob interrupted "the flow" of the conversation. Nonetheless, Nelson felt it was essential to sample a wide range of students to make sure that they participated in the discussion and that they were paying attention. A concern that everyone in the class "got it" was

important for her. In the end, Nelson felt that she had a difficult time finding the right balance between the contributions that would best serve an in-depth mathematical discussion and contributions from a range of students that would insure that everyone was following the lesson.

Putnam and Borko (2000) alluded to this problem recently as a conflict between building a community of learners and the need for individual accountability in classrooms. Applied in the context of this study, Nelson seemed successful in socially scaffolding students into the conventions of classroom discussions by the ninth week. Students were better able to explain their ideas on their own and comment on another student's (or group of students') thinking in a civil manner. In this way and many others, she was able to build a community of learners.

Yet making sure that all of the students in the class were following the discussion and understanding what is being said – the issue of individual accountability – led to an uneven pattern of discourse. The observations and informal interviews suggest that she would have to interrupt the student discussion or abbreviate her probing of a student's thinking to make sure that everyone in the class understood a group's strategy and solution. The academically low achieving students served as unavoidable anchors for Nelson, signaling what the less capable students in the class may have comprehended from the discussion. All of this led to a different kind of discourse than commonly appears in the literature. It was one where the depth of a dialogue or discussion was sacrificed for a breadth (i.e., attempting to make sure that everyone in the class understood something from the presentations). To be sure, trying to insure that 28 students understand the lesson is a considerable task.

At the level of discourse analysis, this first dilemma speaks to the importance of different categories of classroom talk. Discourse research often attends to the content or locutionary meaning of statements, whether they are from the teacher or the student (see Georgakopoulou & Goutsos, 1997). Nelson's continued attempts to discern if many other students in the class understood what was being said highlights the importance of the perlocutionary force of mathematical statements (e.g., did what those in the front of the class say have any effect on the understanding of students who were quietly attending to the discussion). The perlocutionary issues that arose in the reporting out phase provide a theoretical frame for understanding how Nelson's discourse differs from the kinds of classroom discussions that frequently appear in the reform mathematics literature.

The second, and related dilemma has to do with individual differences and the attempt to achieve common intellectual outcomes. It is apparent from the contributions of the academically low achieving students that they are often far less sophisticated than their more capable peers. Furthermore, it is not clear how well they understand the three or four strategies and solutions that they may hear in a lesson. Jacob's persistent concern with the pattern in the numbers, an observation of marginal significance, suggests that he was not attending to the more complex discussion of his group's strategy. Furthermore, a high level of assistance from the instructional aide during small group work also demonstrates that students like Jacob need a significant amount of guidance simply to be able to participate in the lesson.

The differences among the students throughout the study were striking, and reconciling these differences with the goals of the current reform is not a minor affair. Teachers often find themselves positioned between the need to maintain high expectations

for all of their students and the reality of individual differences. When differences in performance are documented in the discourse literature, the explanation is often attributed to cultural factors. Seminal discourse work in the past (e.g., Gee, 1996; Heath, 1983; Walkerdine, 1988) as well as recent mathematics research (Lubienski, 2000) documents the linguistic conflicts between the middle class expectations of schools and socio-cultural backgrounds of working class students.

This explanation is difficult to accept, at least as the primary means of interpreting why the kind of mathematical problem solving in Nelson's class was so difficult for the academically low achieving students. These particular students -- and for that matter, many students with learning disabilities -- do not easily fit into a single socio-economic class category. Instead, it appears that the conceptual and verbal demands of Nelson's class were, at times, well beyond these students. Furthermore, data from the larger case study indicated that their mathematical abilities outside of the weekly problem solving exercises support the notion that the issue has less to do with cultural linguistic issues than academic achievement and ability (Woodward & Baxter, 2001).

While the discourse of academically low achieving students appears to be well behind their more capable peers in these circumstances, it is also difficult to imagine how a pull-out, remedial setting would expose them to a discussion of different strategies and solutions that was student-centered. Given the recent history of intervention research in special education (e.g., Swanson, Hoskyn, & Lee, 1999), it is far more likely that such a discussion would be highly teacher-directed. This kind of structure would clearly conflict with what many in the mathematics reform community have come to value as critical peer-to-peer communication (see Hiebert et al., 1997).

This second dilemma also highlights an important assumption about the movement to improve the rigor of content areas like mathematics. That is, it is assumed that the valued outcomes are almost entirely cognitive. What is evident in Nelson's class is the balance between the social-emotional and the cognitive outcomes of classroom instruction. An inclination to pull students out for highly directed, problem solving instruction of the kind described in this study implicitly privileges cognitive outcomes over the kind of social-emotional concerns that arise from a student's perspective (e.g., a student's desire to stay in class and participate with peers on classroom activities, a student's identification with the regular classroom as the desired learning environment). A striking and consistent finding from the videotape analyses of Nelson's classroom was the number of times the target students volunteered to talk or answer questions. The regular classroom context supported their interest in participating in class discussions, something that by its nature would not have occurred if they had been pulled out during these lessons for remedial instruction. Participating in a community of learners at a social-emotional level while, at the same time exhibiting considerable differences at the cognitive level is sometimes overlooked in the math discourse literature (see Burbules, 1993; Ellsworth, 1989, 1997, for a more in-depth discussion of this topic).

It should be said that these dilemmas do not imply that the kind of intense, individualized instruction that can occur in remedial or special education settings is unimportant or inconsistent with reform mathematics in a broader sense. In fact, the ad hoc tutoring that occurred at other times of the week in Nelson's classroom support the notion that academically low achieving students need highly directed instruction in specific skills and concepts (see Woodward, Monroe, & Baxter, 2001; Woodward & Montague, in press).

At the heart of the issue is how best to develop math problem solving and oral communication in academically low achieving students. Remedial environments that bring together only low achieving students are likely to sponsor little in the way of rich, student-centered classroom discussions. Yet typically sized, regular education classrooms such as Nelson's may not provide the most optimal solution to this problem. Resolving these dilemmas may require further interventions beyond what were provided in this study.

One intervention that could be feasibly added to the efforts described in this study would involve a different kind of ending to the reporting out segment of the lesson. Instead of closing with a discussion of the last group's solution and strategy, the teacher could briefly summarize the two or three strategies described in the reporting out segment. A comparison of strategies or multiple solutions without identifying "the best" solution is integral to reform mathematics discourse.

This could be followed by further, small group work with the aide. Academically low achieving students might review two of the different strategies and then discuss or write about the differences between the methods. This kind of "mini-lesson" would strengthen the link between strategic thinking and oral or written discourse. It would also provide the kind of added, individual attention that at-risk students and those with learning disabilities need in general education classes. In this way, a broader range of students could participate in classwide problem solving sessions, thus making mathematics reform potentially accessible to mainstreamed students with learning disabilities and those students at-risk for special education services.

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Figure 1

Week Three Problem Solving Exercise

Tanya and Lucy were throwing balls into three baskets at the fair. Whoever scored exactly 34 points with the least number of throws won the prize. What is the least number of throws needed to score exactly 34?

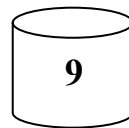
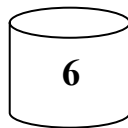
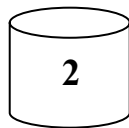


Figure 2

Sample Classroom Dialogue During the Initial Stage

Teacher: "How many throws was that?"

Student: "Throws, that's 5"

Teacher: "Could you hold that up [the white board] so we can see, you're kind of tipping it back. You said 2 sixes and 2 nines, what's the other part?"

Student: "2 sixes, no 2 nines, 3 sixes, which is 18 and 16 which is 34."

Teacher: "2 nines is 18 and 3 sixes is 18"

Student: [realizing mistake, flounders, points to white board] "I think this one works better."

Teacher: "So the one that equals 34 is the one on the right? [addressing the class] From looking at their white board, do you understand their strategy. This is the time to look at their strategy and understand what their doing. Teri, do you understand what they're doing?"

Figure 3

Sample Target Student During the Initial Stage (Week 3)

Teacher: Jacob do you have a solution you'd like to share?

Jacob: I found out that there is more than one different way to get to 34 and so I decided to do it different ways and in the middle I wrote that there is more than one way to get to 34. You just have to add right.

Teacher: How many new teams tried more than one way? Can you tell by looking at Jacob's whiteboard what is the answer to the least number of throws?

Jacob: I decided each group is the group of three. [referring to two sets of 9, 6, and 2 the column of numbers on the whiteboard].

Teacher: Does anyone have a suggestion for Jacob?

Leslie (at her seat): He should have put how many throws.

Teacher: Leslie had an addition number model that she used to solve the problem but she erased it. Do you still have it? Can you hold up in show your number model?

Leslie: Six plus nine equals 15 plus two plus two plus six equals 34.

Teacher: I believe Leslie had another number there and she erased it and didn't get a chance to write it again. You had a six plus nine plus another six plus nine, remember? Once she shared that with her team, they went about trying to explain it. Thanks.

Figure 4

Week Nine Problem Solving Exercise

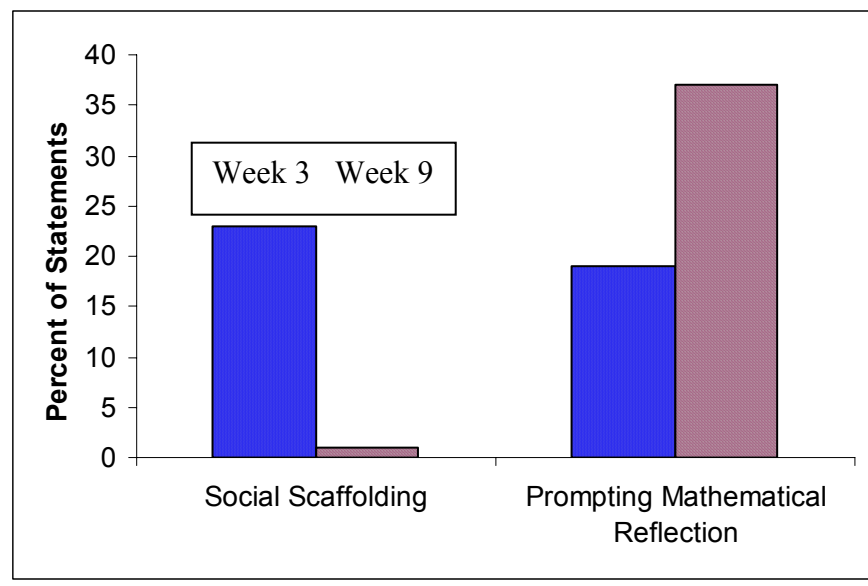
Figure 5**Categorical Shifts in Teacher Statements: Week 3 to Week 9**

Figure 6

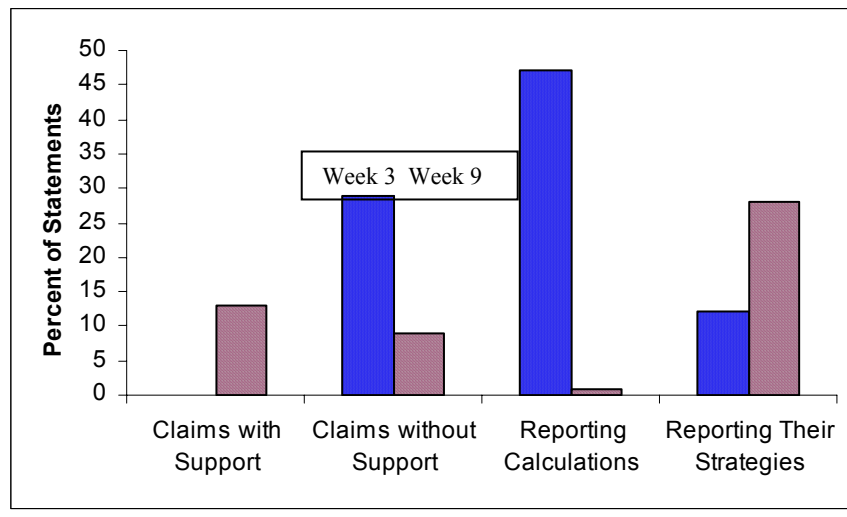
Categorical Shifts in Student Statements: Week 3 to Week 9

Figure 7

Student-Centered Discussion from Mature Stage (Week 9)

Teacher: Let's talk about the answer for a minute. Tom, can you find the answer to your solution?

Tom: Its says 10 chairs and 5 stools.

Teacher: And do we agree that's the right answer?

Several students in the class: Yes

Teacher: And what about the way they communicated their strategies? Can you follow their strategies? Can you understand? How about you Leslie? [Leslie has her hand raised.]

Leslie(TAR)¹: I was wondering about, I wonder where they got the 15.

Teacher: Let's ask them some of those questions. Can we start with Leslie's question? Can you tell Leslie the answer to her question?

Teacher: I don't think they heard your question. Can you ask it again?

Student presenting in the front of the room: We were guessing numbers at the top and 10 times 4 equals 40, so we did 55 minus 40 equals 15.

Teacher: Hector? (student in the class)

Hector: Why did you do 10 times 40? I mean 10 times 4.

Kyle (presenting in the front of the room): We were guessing numbers.

Another student in the class: Then did you do any mathematical thinking? Does mathematical thinking give you 15 when you use guess and check?

Kyle: Yes, the legs for the chair is 4, so we did 10 times 4 and we get 40 so we did 55 minus 40 and got 15 so that would help us get our answer for the bottom that was 10, no 5 stools, so we did 10 chairs and 5 stools.

Hector: You could've gotten 5 times 4 or 8 times 4 or 9 times 4

Kyle: We get 15 divided by 3 because there are 3 legs in a stool.

Teacher: Hector was wondering if your first guess was to choose 10 chairs?

Kyle: We have it over there [referring to the paper they used to work the problem], we have a lot guesses on that side.

Teacher: I think the point that the class is making is that they can't tell that guess and check was your strategy because there aren't any guesses there.

Kyle: But me and Carrie discussed that and we couldn't fit everything on there [the overhead transparency] so that was the right one [answer] that we put up.

¹ Denotes target student

Figure 8

Reproduction of the Overhead that Jacob's Group Used in the Week 9 Lesson

4.
4.
4.
4.
4.
3°
4.
3°
4
3°
4.
3°
4.
3°
4.

$$\begin{array}{r} \underline{10} \times 4 = 40 \\ \underline{5} \times 3 = 15 \\ \hline 55 \end{array}$$

So the answer is
10 chairs and
5 stools

Figure 9

Target Student's Talk During the Week 9 Lesson

Sharon: We got the 10 times 4 and the 5 times 3 because the story problem said that there were 5 less stools than chairs. The list has 5 fours and it has threes and fours and we counted up all of fours and 10 times 4 is 40 and we counted up the threes and 3 times 5 is 15 and we added 40 and 15 and we got the 55 and that's the number of legs.

Teacher: Did you follow Sharon's strategy? Anyone else from Team 2, do you want to say anything else about your explanation? Jacob?

Jacob: The reason why in our list we don't have a number on the bottom is that we're just seeing if we could, we're just writing ten fours and five threes and it wasn't how we figured out that the answer was ten fours and five threes. That's how we found out that the answer was ten and five so then we could know how many of fours and threes and then fives.

Teacher: Now that I know that you were making a list, I see why you didn't put a plus or a times sign.

Student presenting in the front of the room: Yes, we weren't trying to add them up. We were just trying to count them, this just made it easier.

Teacher: So your number model shows the conclusion to your list and I understand now that the list is organized.

[2 minutes later]

Teacher: First, I'd like to have them answer about the order. Sharon did explain it, but it sounds like it was kind of missed by you so I'm going to ask someone else at your table. Savrina, why is the list in that order? What made you start with all those fours?...Why does it start with five fours, why not five threes?

Jacob [volunteering]: Maybe that's because we had to have five fours at the top and five fours at the bottom.

Teacher: Is there reason for putting those fours of there? I heard Haley tell us all why it is that way. There was a reason for making a list in that order. What could their team have done to let us know what their strategy was, and why they put the numbers in that order? Can you give them one suggestion that would help there's be the strongest it could be?

[1 minute later]

Teacher: Jacob, do you want to say one last thing?

Jacob: [with hand raised] The reason why we put five fours at the top was to make it even so that we would have five fours at the bottom.

Teacher: I don't see five fours at the bottom

Jacob: [points to each four in the bottom half of the list and counts] 1,2,3, 4,5 we just made it that way to make it even on the top and the bottom.