

Meeting the Challenge of Mathematics Reform For Students With Learning Disabilities

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The ideas reported in this manuscript have been supported by a grant from the U. S. Department of Education (H023V70008) to the REACH project (Research Institute to Accelerate Content Learning through High Support for Students with Disabilities in Grades 4-8).

Abstract

The purpose of this article was to examine the sustained effort to reform K-12 mathematics instruction in this country over the last 10 years and the implications of this reform for students with learning disabilities. We began by describing 3 forces that have driven mathematics reform: shifting theoretical paradigms, disappointing levels of mathematics performance of students in the United States, and the impact of rapidly changing technologies. Then we discussed concerns about this reform from the special education community. In the second half of the article, a synthesized special education research relevant to mathematics reform was provided along with thoughts about future directions in mathematics education for students with learning disabilities.

Meeting the Challenge of Mathematics Reform For Students With Learning Disabilities

The *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000) reflects a decade of sustained reform in mathematics. The first version of the *Standards* (i.e., the *Curriculum and Evaluation Standards for School Mathematics*, NCTM, 1989) was the impetus for many curricular and pedagogical changes that have occurred since its publication. Arguably, the 2000 *Standards* remain as the centerpiece of mathematics reform in the United States. Both sets of *Standards* reflect a high level of consensus around longstanding efforts for educational reform not only in the United States, but internationally as well. All major industrialized countries in the world have been trying to reform their mathematics pedagogy and curriculum in significant ways since the late 1980s (see Atkin & Black, 1997; Australian Educational Council, 1990; De Corte, Greer, & Verschaffel, 1996; Schmidt, McKnight, & Raizen, 1997; Treffers, De Moor, & Feijs, 1989).

From the outset, the 1989 *NCTM Standards* generated a great deal of interest and controversy in both general and special education communities. To be sure, the agenda set out in the 1989 *Standards* was ambitious. The call to develop children's "mathematical power" meant that *all* children needed to acquire a new level of conceptual understanding and to be able to engage in ongoing discourse about mathematics as they solved challenging problems. It was expected these cognitive outcomes could be achieved through demonstrative changes in curriculum and pedagogy. In fact, the 1989 *Standards* suggested that new forms of pedagogy, ones where teachers facilitated discussions and probed children for alternative ways of thinking, would be central to the reform.

One of the major problems with the 1989 *Standards* was that they did not specify how the call for uniformly high outcomes for students at a national level would be reconciled with the varying needs inherent in serving a large and diverse population of students. This problem was acutely apparent to the special education community, as students with disabilities were neglected throughout the document. There was virtually no mention in the 1989 *Standards* how the suggested curricular and pedagogical reforms would affect students with disabilities, the majority of whom were or would be receiving mathematics instruction in general education mathematics classes. At a practical level, both special and general educators were concerned about how these major shifts in curriculum and teaching methods would occur given the limited resources available to enact them.

Despite the document's vagueness about implementation and diversity, there was enthusiastic support for the reform across many groups in the 1990s, particularly among policy makers. States began investing considerable money and effort in curriculum development and teacher inservice to facilitate acquisition of the necessary pedagogy to implement a new curriculum. By 1993, 41 states had taken action to adopt the 1989 *NCTM Standards*. As of 2001, all but one state had new content or curriculum standards in mathematics.

In order to probe current thinking about mathematics reform and its implications for students with learning disabilities, it is important to go beyond the *Standards* themselves. Too often, broad criticisms of the *Standards*, many of which are reviewed in this article, belie a specific concern about pedagogical methods. Disentangling some of the different forces behind the *Standards* help elucidate the promise and the problems special educators see in contemporary math reform.

In the first half of the article, we examined three of the more important forces that have shaped mathematics reform over the last 30 years: shifts in theoretical paradigms, disappointing levels of mathematical performance of students in this country, and broad advancements in technology. We then provide an analysis of special education's response to the *Standards* over the last decade. In the second half of the article, we described current special education research and promising practices that are in keeping with the 2000 *NCTM Standards* and should facilitate mathematics instruction for students with learning disabilities in the context of curricular and pedagogical reform.

Forces Shaping Mathematics Reform

Shifts in Theoretical Paradigms

Major changes in theories of learning have been as apparent in mathematics education as they have been in other disciplines. Behaviorism was the prevailing theory of learning and instruction during the 1950s and 1960s. When cognitive psychology reemerged in the late 1960s and early 1970s, researchers in both general and special education began to consider cognitive explanations of learning and incorporate these notions into their research programs. Cognitive psychology departed from the strict orthodoxy of behaviorism and, with its several interrelated branches, began to affect educational research and practice. Neo-Piagetians (e.g., Case, 1985; Flavell, Miller, & Miller, 1993), information processing theorists (e.g., Simon, 1989; Sternberg, 2000), and socio-cultural theory (e.g., Vygotsky, 1989, Wertsch, 1991) continue to be major influences on research and practice. Intervention researchers, however, recognized the utility of principles like task analysis and, therefore, did not abandon behaviorism entirely. This

trend toward eclecticism in theoretical foundations for designing instruction was especially apparent in special education (Gersten & Baker, 1996; Swanson, 1988, 1989).

The *NCTM Standards* seem to be grounded in cognitive and constructivist approaches to learning as reflected in an emphasis on developing children's ability to think about mathematics. In order to accomplish this, the Standards advocate numerous changes in content, pedagogy, learning tasks and experiences, and the context for learning, curriculum, and evaluation. Six principles guide educators in their attempts to reform mathematics instruction: (a) "equity," which implies high expectations and instructional support for all students, (b) a well-articulated and coordinated "curriculum," (c) effective "teaching" that demonstrates an understanding of students' knowledge and then provides challenging activities to encourage new learning, (d) "learning," which implies the active construction of mathematical knowledge by students, (e) "assessment" that supports learning and is useful to students and teachers, and (f) "technology" as essential in mathematical instruction and learning.

At the core of the constructivist agenda are assertions that learning is an active, social, and interactive process; that learners construct an understanding of subject matter rather than copying it directly from the teacher; and that authentic activities are the foundation for the construction of knowledge (Bransford, et al., 1999; Hiebert, et al., 1997). This approach considers learners' misunderstandings or misconceptions as important learning opportunities. As fundamental as these tenets may appear, over the last 20 years, various positions on constructivism have emerged. This has complicated a clear understanding of this theoretical approach to instruction, leaving it ill-defined in the minds of many (Prawat, 1999). One can find constructivist literature that focuses predominately on the individual (Glaserfeld, 1991), on the social or sociocultural (Forman, Minick, &

Stone, 1993; Palincsar, 1998), and even on the political or “emancipatory” (Vadeboncoeur, 1997).

Both the individual and the social perspectives are widely apparent in the general education mathematics literature. Von Glaserfeld’s (1995) radical constructivism, for example, places a high value on *individual* constructions or inventions of mathematical knowledge. He suggests that there are limits to how much shared meaning exists between students about a given concept and that a student’s misconception cannot simply be replaced by the teacher’s “right way of looking at things.”

In contrast, the *social* constructivist perspective highlights classroom community and discourse. Making common mathematic topics such as two digit addition or subtraction problematic and treating them as “ill-defined” are recurrent themes in this strand of the reform literature, as a means to get students to “think mathematically” (Ball, 1993; Cobb, Wood, & Yackel, 1993; Hiebert et al., 1997; Lampert, 1990; Lampert, Rittenhouse, & Crumbaugh, 1996; Resnick, 1988). As Geary (1994) noted, this view, if taken to an extreme, makes mathematics almost entirely a social enterprise, where discussion and disagreement are the basis for a student’s conceptualization and re-conceptualization of key concepts.

Social constructivist thinking can also be found in studies that emanate from cultural psychology. Research into the mathematical practices of street vendors (Nunes, Schliemann, & Carraher, 1993), tribal groups (Saxe, 1992), and grocery store shoppers (Lave, 1988) offers a sociohistoric or “situated cognition” perspective. One theme from this ethnomathematical literature is that even though individuals may perform poorly on standardized or formal measures of mathematical understanding, they can demonstrate high levels of competence in practical, everyday settings. Another more obvious message from

the ethnomathematical literature is that an individual's culture can have a profound effect on the development of her or his informal mathematics knowledge.

The striking variations among these views of constructivism make a single account of the theory problematic. As Richardson (1997) observed, constructivism offers no one, simple perspective on teaching. For these reasons, and because special education is so grounded in behavioral theory, many special educators have understandably responded with caution to the shift toward cognitive and constructivist views of teaching and learning (e.g., Gersten & Baker, 1998; Harris & Graham, 1994, 1996; Mercer, Jordan, & Miller, 1994).

There is a history in special education research of placing a considerable emphasis on rote learning and mastery of math facts and algorithms for basic operations (e.g., addition, multiplication) and limited instruction in problem solving (see Swanson, Hoskyn, & Lee, 1999). Several researchers have noted that even though traditional methods such as direct instruction may be effective for teaching factual content, "there is less evidence that this instruction transfers to higher order cognitive skills such as reasoning and problem solving" (Palincsar, 1998, p. 347). Given the longstanding need to access to general education curriculum for students with disabilities, special educators must reconsider traditional approaches and provide instruction that is more consistent with the reform agenda.

Disappointing Levels of Mathematical Performance

General education also has been described as overemphasizing rote instruction and didactic teaching. Criticism of the highly procedural nature of mathematics instruction in the United States preceded the "new math" efforts of the 1960s. An array of textbooks, commentaries, and research from the 1940s and 50s (e.g., Brownell, 1945; Marks, Purdy,

& Kinney, 1958; Wertheimer, 1959) contained critiques of instruction that focused only on rote mastery of arithmetic knowledge. Similar criticisms of instruction in mathematical concepts and problem solving appeared in the 1980s (see Grouws, 1992). The basis of the argument was that instruction was unsatisfactory because it concentrated almost exclusively on the mastery of algorithms used to perform operations on whole and rational numbers. As a result, students failed to achieve a sufficient conceptual understanding of the core concepts that underlie those operations and algorithms (Baroody & Hume, 1991; Hiebert, 1986; Hiebert & Behr, 1988; Skemp, 1987).

Another concern was that problem solving instruction typically involved only problems that could be solved in “five minutes or less” (e.g., Doyle, 1988; Schoenfeld, 1988). This type of rapid and artificial problem solving is most apparent when students are taught to search for key words (e.g., “more” or “gave away”) or use a single strategy (e.g., “make a table”) to solve a set of predictable or highly structured problems. Such an approach may enable students to complete what has been called “end of the chapter problems.” However, critics argued that these methods do little to foster a deeper sense of problem solving. In fact, they may lead many students to give up when faced with more complex problems that require extended effort or several incorrect attempts before a correct solution is achieved. Yet, complex problems reflect the kind of effort required in authentic problem solving and more sophisticated mathematics.

Results from international studies of students' mathematical proficiency over the last two decades have been disappointing for the United States, thus reinforcing educators' concerns. The most recent studies (e.g., Third International Mathematics and Science Study or TIMSS; Beatty, 1997) and subsequent TIMSS-Repeat (TIMSS, 1999), each involving about 40 countries, confirmed that students in the United States are not performing as well

as students in many other developed countries, especially Asian countries. In 1999, U.S. students were about in the middle as compared with 38 participating countries. The top performing countries were Singapore, Republic of Korea, Chinese Taipei, Hong Kong, Japan, and Belgium. As Schmidt and his colleagues (Schmidt et al., 1997) suggested, mathematics instruction in the United States suffers from a “splintered vision,” with curricula that focus on too many superficially taught topics in a given year. More successful approaches, found particularly in Asian countries, tend to focus on fewer topics. The lessons are often devoted to the analysis of a few examples, and teachers encourage students to share different solutions to problems (Office of Educational Research and Improvement, 1998; Stevenson & Stigler, 1992; Stigler & Hiebert, 1999). Most certainly, there are educators (e.g., Berliner & Biddle, 1995; Bracey, 1997) who disagree with the thrust of these criticisms. However, their commentaries tend to address longstanding and broad based critiques of American education rather than problems with mathematics education per se.

Technological Advances

The broad impact of technological advancements over the past 30 years has been startling and well-documented. Much has been made of our current economic transformation from an industrial society to an information economy (Davis & Maher, 1996; Greider, 1997; Reich, 1991). It is ironic, however, that with constant technological change all around us, it is difficult to appreciate either the near or long term effects of these changes. In computing, the micro processing power has doubled every 18 to 24 months since the late 1950s (i.e., Moore’s Law) and is expected to continue well into the first two decades of the 21st century (Patterson, 1997). The doubling of processing power to this point in time has been impressive but not overwhelming at the level of desktop computers.

Continued doubling in processing power over the next 20 years coupled with declining costs will create personal desktop computing systems on *frequent intervals* of entirely different magnitudes than we have ever experienced. Over the next decade, personal computers will approach supercomputing capacities.

At the other end of the spectrum, we are entering a profoundly important era of embedded computing. The term, *embedded computing*, refers to an ever-growing array of inexpensive computing devices found in everyday appliances and tools. No longer will hand calculators present the “revolutionary” promise for mathematics instruction as Romberg (1992) predicted. Rather, a variety of hand-held devices that perform important computational and communicative functions will become increasingly available.

The range of computing tools, from hand held devices to powerful computers, will increasingly be used to perform the very computational functions that they have practiced on a daily basis by hand. These tools already allow today's student (and workers) to operate on data at a much higher level (e.g., analyze it for trends, perform statistical operations, solve complex problems).

These technological trends have considerable implications for students with learning disabilities. For students who continue to learn a narrow range of mathematics this will be a growing problem (e.g., instruction that focuses largely on paper and pencil mastery of computational problems). Continued advances in technology will only accentuate the gap between what is typically taught to students with learning disabilities and what individuals need to know in a world filled with computing devices (see Goldman et al., 1997).

Special Education, Mathematics Reform, and the *NCTM Standards*

The first source of criticism of mathematics reform is the shift from traditional didactic models of instruction to constructivism. Schools are increasingly adopting mathematics programs such as *Everyday Mathematics* and the *Connected Mathematics Program* that embrace constructivist approaches. To exemplify, Tarver (1996) argues that constructivism is tantamount to discovery learning and, as a pedagogical approach, most likely will lead to even greater failure for students with learning disabilities. She ardently defends direct instruction as the most effective approach to education for these students. Her position stands as a marked counterpoint to the trend toward full inclusion for students with disabilities, IDEA's requirement to provide access to the general education curriculum for these students, and the increasing emphasis on developing students' mathematical "power."

In contrast, others have argued that traditional special education instructional methods and constructivism are more compatible than not. They have made a conscious effort to distinguish between the kind of constructivism that stresses child-determined, guided discovery and a more structured variant that provides skill development and guided practice (Harris & Graham, 1994; Mercer, Jordan, & Miller, 1994). This view maintains that constructivism does not preclude skills instruction and development. Bransford et al. (1999) addressed this by noting a "common misconception regarding 'constructivist' theories of knowing (that existing knowledge is used to build new knowledge) is that teachers should never tell students anything directly but, instead, should allow them to construct knowledge for themselves. This perspective confuses a theory of pedagogy (teaching) with a theory of knowing (p. 11)." Even von Glaserfeld (1991), who is regarded by some as having the most extreme epistemology views on constructivism, acknowledged

that memorization and rote learning are unavoidable in education. Special educators' efforts to ensure a place for skills instruction in a constructivist framework are both important and understandable when faced with the educational needs of students.

Recently, Gersten and Baker (1998) attempted to merge constructivist and behavioral models of instruction in their effort to link direct instruction with situated cognition. They suggested behavioral methods like direct instruction provide the necessary skill base for problem solving and are easily incorporated into routines and approaches such as anchored instruction. Bottge and Hasselbring's study (1993) controlled for students' prior knowledge using a direct instruction math videodisc program to develop a common knowledge base in fractions before implementing the intervention. Based on this preteaching, they were able to contrast the effects of anchored instruction problems with more traditional problem solving methods and avoid important confounds. A more accurate view of Bottge and Hasselbring's anchored instruction research, as least as it applies to practitioners, is that skills development should be embedded in instruction as needed (Goldman, et al., 1997; Hasselbring, personal communication). Reid (1998) made a similar observation in her discussion of how some special educators misinterpret scaffolding and its application in practice. Concepts such as scaffolding, situated cognition, and direct instruction emanate from different theories of learning, motivation, and instruction (see Greeno, Collins, & Resnick, 1996).

Pedagogically, it is difficult to see how a teacher would provide teacher-directed, step-by-step skill instruction for a lengthy period of time and then shift frameworks and employ interactive scaffolded instruction, inquiry about misconceptions, and authentic tasks as Gersten and Baker (1998) suggest. Extended, hierarchical skills instruction in an area such as fractions – instruction that places little emphasis on conceptual understanding -

- only then to be followed by anchored problem solving is conceptually problematic.

Woodward, Baxter, and Robinson's (1999) recent study of direct instruction methods in decimals and percents delineates the problems of trying to provide a "skills-only" foundation in a mathematical topic before moving on to conceptual understanding or problem solving. They found that students who were taught with direct instruction methods for four weeks were ill-prepared for more conceptually-based instruction in the subject. A more disturbing finding from this study was that retention of the skills mastered during the instructional intervention quickly atrophied.

The second main source of criticism from special educators has to do with the *NCTM Standards* themselves. The *NCTM Standards* have been described as elitist (Hofmeister, 1993), too difficult to implement (Carnine, Jitendra, & Silbert, 1997), devoid of a research foundation (Carnine, 1992; Carnine, Jones, & Dixon, 1994; Kameenui, Chard, & Carnine, 1996; Stein & Carnine, 1999), and representative of discovery-oriented constructivism (Mercer et al., 1994). Even though many of the criticisms are polemical, important concerns and common misconceptions about the *NCTM Standards* are apparent. For example, some of the difficulty special educators have with the *NCTM Standards* may reside in way they are written. The *NCTM Standards* are not intended as a detailed series of objectives for daily instruction nor as an academic research document. Rather, they are written for a broad audience of practitioners, policy makers, and those interested in educational reform. In this regard, they are in keeping with previous publications for diverse audiences (e.g., *Everybody Counts*, National Research Council, 1989).

The claim that the *NCTM Standards* lack a research foundation appears, on the surface, as extraordinary given research syntheses such as Grouw's (1992) *Handbook of Research on Mathematics Teaching and Learning* as well as many other publications over

the last two decades (see De Corte et al., 1996). At a more subtle level, however, this concern over the presumed lack of adequate research support generates a more substantive question, one that is typically unstated in the critics' polemics. Hiebert (1999) offers a thoughtful answer to this question in his recent discussion of the relationship between research and standards in any field. He observes that for several reasons, research cannot begin to account fully for all aspects of standards for a field.

One central reason for this is that research support is *likely* to be uneven when standards are developed. Standards, after all, are policy documents. They are intended to promote further research in a given area as much as to reflect what is already known. Second, and perhaps most importantly, standards reflect what is *valued* in education. This observation is crucial in any consideration of the mathematics for students with learning disabilities. Narrow attempts to define "what counts" as valid research (e.g., Dixon, Carnine, Lee, Wallin, & Chard, 1998) miss these two points. Putting aside the quality of the Dixon et al. report (see Becker & Jacob, 2000), it relies on research design criteria that result in an atheoretical collection of studies that offer little, if any, *direction* for math education. They do little to achieve the kind of convergent understanding needed to move a discipline forward. This view is reinforced throughout the recent *Handbook of Research Design in Mathematics and Science* (Kelly & Lesh, 2000). Addressing what is valued in education is a relatively recent stance for those concerned with curriculum design and classroom pedagogy. As Greeno et al. (1996) claimed, there is an all too common tendency to take what generally appears in commercial materials and find ways to teach that content to students more efficiently, rather than to address more basic questions of what is worth knowing.

Reformers like Hiebert (1999) also noted that the traditional methods for teaching mathematics in the United States have done little to help us meet the kind of rigorous outcomes described in the *NCTM Standards* nor have they led to improved standings on international tests such as the Third International Mathematics and Science Study. Stigler and Hiebert's (1999) recent analysis of American, German, and Japanese curricula and classroom pedagogy detail the problems with the kinds of process-product approaches to mathematics where students concentrate on a mastery of procedures with only a marginal focus on conceptual understanding. As mentioned earlier, these observations have direct implications for intervention research in special education. Specifically, they argue that protracted instruction in skills and procedures without a balanced emphasis on conceptual understanding and problem solving is problematic.

In summary, the two main sources of concern for special educators (i.e., constructivism and the lack of a research base for the *NCTM Standards*) are much more complex than commonly thought. Undoubtedly, many in the field will always find constructivism troubling, even with a well-articulated skills component. This is understandable given the disconnection between constructivism and an “embedded skills” in mathematics which is new and rarely articulated for students with learning disabilities. Nonetheless, it should be clear that the two other forces guiding math reform -- the longstanding critique of mathematics and the continued evolution of technology -- argue for a serious re-examination of common mathematics practices for students with learning disabilities.

NCTM Standards and Instruction for Students with Learning Disabilities

This section describes new directions in mathematics research that seem in keeping with the *NCTM Standards* and also appear to hold particular promise for improving performance in math fact acquisition, computation, and problem solving for students with learning disabilities. Some of these directions have a substantial research base, while others are emerging but hold promise for future research.

It should be noted that this section is not an exhaustive attempt to discuss how mathematics topics should be adapted for students with learning disabilities. In fact, there are many areas of mathematics (e.g., measurement, geometry, simple levels of probability and statistics) where there is little research involving students with learning disabilities. The relative dearth of research in mathematics for students with learning disabilities is apparent in recent meta-analyses of intervention research (see Swanson et al., 1999). This section also does not attempt to describe how students with learning disabilities should be taught mathematics using a small set of generic principles of curriculum design or pedagogy. We feel that approach understates the subtle but critical ways in which content issues in a domain guide instruction.

One constraint that must be mentioned in any discussion of improving practice for students with learning disabilities is the issue of limited instructional time. Time constraints have an impact on what, how much, and when students learn. Too often in special education, students are shortchanged in mathematics instruction, particularly higher level mathematics, because they have so many other pressing needs including, not limited to academics. Such time constraints force us to examine carefully what we want students to learn and how this will affect educational outcomes. Rather than a more efficient reworking

of traditional mathematics using a common hierarchy of skills, outcomes need to be considered in light of promising directions in recent mathematics research (see Goldman et al., 1997 for a further discussion of this issue).

Instruction in Math Facts

Information processing approaches to math instruction for students with learning disabilities emphasize the importance of fluency in fact retrieval. The argument is that quick and efficient math fact recall or automaticity enables students to devote more of their cognitive resources to the procedural knowledge associated with learning algorithms (Gerber, Semmel, & Semmel, 1994; Pellegrino & Goldman, 1987). Several studies suggest that students with learning disabilities tend to use immature strategies when they learn math facts. Goldman, Pellegrino, and Mertz (1988), for example, found that students with learning disabilities were well behind their peers in using *min* counting (e.g., adding $2 + 9$ as $9 + 2$) and direct retrieval strategies for addition facts. Students with learning disabilities tended to use counting all strategies (i.e., laborious one-by-one counting to achieve answers) even after extended practice.

Putnam, deBettencourt, and Leinhardt (1990) found a similar tendency for immature strategies when students with learning disabilities were taught to use derived fact strategies for retrieving facts such as doubling (e.g., $6 + 7 = 6 + 6 + 1$), sharing ($7 + 9 = 8 + 8$), and going through ten ($9 + 4 = 10 + 3$). Although this work focused less on automaticity than Goldman's et al. (1988) and more on part-whole conceptual understanding, both researchers concluded that the performance of students with learning disabilities was *delayed* and not cognitively different from normal achieving children. In this regard, their findings were consistent with earlier research (Russell & Ginsburg, 1984).

Research suggests that at least a subset of remedial students and those students with disabilities (specifically, math dyscalculia) might have immature strategies such as counting all, but whose actual difficulties are much more profound. Geary's (1994) summary of a number of related studies on this issue focuses on two concerns. First, his research with first grade, in particular, found that about half of the children were misidentified for remedial and special education services and did not show any form of a cognitive deficit (Geary, 1990; Geary, Bow-Thomas, & Yao, 1992). The remainder of students, however, had significant problems representing facts in long-term memory and were highly inconsistent in the speed at which they retrieved these facts. For these children, Geary argued that retrieval is not likely to improve, at least without extensive remedial training. In his view, students with learning disabilities are categorically different from those who exhibit developmental delays.

A comprehensive accounting of the difficulties these students face in learning math facts is complicated by the structure of facts themselves. That is, addition and subtraction lend themselves to a variety of strategies (e.g., min, doubling) that do not work for multiplication and division. The difficulties in learning multiplication facts are due to the way facts may typically be learned and then stored in associative memory.

Dehaene (1997) observed that the complex network of associations for multiplication facts in conjunction with less practice on some of the more complex facts (e.g., 9×6 , 7×8) leads to confusion and recall errors. Research-based techniques for teaching automaticity are rooted in information processing theory, which suggests methods that have general application for teaching a wide array of declarative knowledge.

Hasselbring, Goin, and Bransford's (1988) methods, for example, delineate the importance of pretesting students, introducing new facts in small sets, providing systematic

review, and controlling response time as ways of moving students from counting all strategies to direct retrieval. Others (Cybriwsky & Schuster, 1990; Kosciński & Gast, 1993) enhance this model by stressing the importance of recycling “known” facts to provide added practice and to motivate students in limited drill and practice exercises. Yet, as Putman et al.’s (1990) research suggests, automaticity is only one aspect of competence in math facts. Teachers need to link facts to a broader network of mathematical knowledge. This entails attention to number sense or “decomposition strategies” and how facts relate to everyday contexts. This observation is critical because the way in which facts are used in mathematics has changed. Math facts seem to play a greater role in mental computation and estimation, both of which rely on number sense skills, than they do in paper and pencil computations.

Jones, Thornton, and Toohey (1985) demonstrated the potential for developing number sense in students with learning disabilities by teaching fact strategies such as doubling, sharing, and “going through ten.” For example, students are taught facts such as $8 + 5$ as $8 + 2 + 3$ or $10 + 3$. This is one example of a math fact strategy that develops number sense in children, which potentially has a broader application in mathematics learning. Woodward and Eckholds (1999) examined the extension of facts into number sense in a year-long study of academically low achieving first graders. These students had initial difficulty extending their knowledge of simple addition facts to larger quantities (e.g., $3 + 2 = 5$, $30 + 20 = 50$). However, after sustained practice, these students showed dramatic improvement in this skill and also improved in understanding the principle of commutation in addition.

Instruction in Computations

Teaching students traditional algorithms for performing the four basic operations in mathematics (e.g., regrouping in subtraction or dividing decimal numbers) is one of the most common instructional activities for students with learning disabilities. Observations of special education classrooms indicate that this type of skill instruction dominates (Parmar & Cawley, 1991, 1995; Rieth, Bahr, Okolo, Polsgrove, & Eckert, 1988). Computational practice is usually structured hierarchically, with an emphasis on mastery of procedural steps in an algorithm as students move from easy to complex problems. Students frequently learn these rote procedures without any conceptual understanding. The logic of computational drill and practice, it seems, is that paper and pencil proficiency is a highly valued aspect of mathematics instruction. It is also the kind of instruction that has considerable “face validity” with many practitioners. Early versions of curriculum-based measurement (Fuchs, Fuchs, & Fernstrom, 1992; Fuchs, Fuchs, & Fernstrom, 1993) as well as the continued nature of direct instruction interventions (Carnine, Jitendra, & Silbert, 1997; Howell, Sidorenko, & Jurica, 1987; Kelly, Gersten, & Carnine, 1990; Stein, Silbert, & Carnine, 1997) exemplify the emphasis on computational drill and practice.

However, spending inordinate amounts of time on computational drill and practice may not be all that beneficial for students with learning disabilities. First, achieving mastery does not come easily, as many information processing studies of computational errors or “bugs” have suggested (Van Lehn, 1990; Woodward et al., 1999; Woodward & Howard, 1994; Woodward, Howard, & Battle, 1997). The likelihood that a student will make some kind of error is evident in a problem such as 357×43 . To achieve a solution, the student must perform as many as 30 correct operations (e.g., retrieving multiplication and addition facts accurately, writing digits correctly and in the proper location, using

scratch marks above the top number correctly, mentally adding scratch marks to multiplication facts). A mistake in any one of these 30 operations most likely produces an incorrect answer.

Some researchers (e.g., Geary, 1994) argue that the problems that academically low achieving students exhibit as they attempt to master computational algorithms are developmental. Eventually, it is believed, the vast majority of these students will become proficient. Yet, an extensive analysis of worksheets from over 400 middle school students with learning disabilities from three school districts suggests that the majority of students still had not mastered algorithms for a beginning operation like subtraction, a skill that most of the students had been practicing for five or six years (Woodward & Howard, 1994). Lack of mastery may be attributed to the highly procedural nature of the instruction that occurs in special education classrooms (Parmar & Cawley, 1991). Without a substantive and persistent link to the conceptual underpinnings of these algorithms, the chances of error increase considerably (Hasselbring, Bottge, & Goin, 1992; Hiebert, 1986).

It should be noted that the nature of the algorithms typically taught may contribute to the students' difficulties. Many educators do not realize that the algorithms they teach evolved historically and that they are not used universally in countries around the world today. During the late 15th century when commerce was becoming important, many algorithms were redesigned for more efficient and faster computation (Swetz, 1987). As Nickerson (1988) noted, those changes resulted in algorithms that were less conceptual in nature. Many math reformers now strongly recommend that students learn a variety of algorithms for performing the same operations, including those that make the conceptual aspects of the operation and the role of place value more explicit (see Morrow & Kenney,

1998). Yet whether or not students with learning disabilities would benefit from multiple algorithms or simply more conceptual algorithms remains an important point of research.

The major question for students with learning disabilities remains. What is the most worthwhile use of limited instructional time for these students? It is not at all evident that they need to be able to solve problems such as 357×43 by hand. Such tasks are best suited to readily available technologies such as calculators. Replacing extensive paper and pencil practice with the thoughtful use of calculators is certainly a key component of mathematics reform (Romberg, 1992; Usiskin, 1998), and limited research indicates that special education students benefit from calculator practice (Horton, Lovitt, & White, 1992; Woodward et al., 1999). What remains for many practitioners and researchers, however, is the question of exactly how calculators should complement or substitute for paper and pencil practice.

One way to shift toward a thoughtful use of calculators is to focus on the kind of conceptually-guided instruction that is now advocated in the math reform literature and the *NCTM Standards*. Over the last 15 years, conceptually guided instruction has moved from a traditional cognitive and developmental orientation (e.g., Hiebert, 1986; Skemp, 1987) to a social constructivist position (e.g., Ball, 1993; Cobb, 1999). Some special education math researchers and curriculum developers (e.g., Mercer & Mercer, 1997) have outlined an instructional agenda in which students with learning disabilities are taught a range of algorithms for basic whole number operations. The partial products algorithm, for example, makes explicit the place values in a problem like 362×4 . Lampert (1986), whose work exemplifies social constructivist practice, draws on algorithms such as partial products as a way to enhance conceptual understanding of multiplication.

The focus of this approach to instruction is on conceptual understanding of key topics (e.g., regrouping, place value) as well as the integrated use of calculators. Through carefully scaffolded activities using authentic tasks, students with learning disabilities could study alternative algorithms or contrast, for example, partial product methods with the traditional multiplication algorithm (see Hiebert et al., 1997). Thus, problem solving and analysis replace drill and practice, and calculators replace paper and pencil computation. Again, calculators can play an integral role in freeing students from the mechanics of learning how to operate on rational numbers (e.g., divide decimals).

A conceptually-guided approach would also enable skill development in the area of number sense. Teachers would be in a position to focus on mental computations and estimation, aspects of number sense that follow directly from this new approach to computations. Thus, students would learn how to “round and multiply” problems like 357×43 into one of many forms based on the context. A context in which a liberal estimate was appropriate might allow the student to compute 400×40 . This could be done mentally or quickly by hand, though the paper and pencil algorithm might be different from the traditional one that is currently taught to students. A different context requiring closer approximations may necessitate a more elaborate strategy (e.g., $350 \times 40 = 300 \times 40 + 50 \times 40$). Clearly, these skills develop over a considerable period of time. However, the knowledge gained would be far closer to what we value today in respect to competence in mathematics. At a mundane yet significant level, this knowledge is critical to assessing whether or not an answer on a calculator screen is correct or the result of improper keying.

This kind of an approach to computation would enable teachers to concentrate more on the topics in which whole number operations are applied. Baroody and Hume (1991) described multiple representations for teaching fractions and how they can be used with

students with learning disabilities. Much of this instruction is consistent with the kind of conceptually-guided teaching described in the social constructivist literature.

These suggestions for reconceptualizing the way students with learning disabilities “manipulate numbers” (e.g., focus more on the conceptual side of basic operations, develop multiple strategies for approximating numbers) most certainly require further research. While they are consistent with the *Standards*, they are also consistent with Hiebert (1999) commentary on the *Standards*. That is, not all *Standards*-related practices have a substantive research base at this point in time. Furthermore, these suggestions also remind us of the tradeoffs that are likely to occur with students like ones with learning disabilities simply take more time to acquire mathematical knowledge. There comes a point in a student’s education, say the beginning of middle school, when educators need to take into account the limited time remaining for the formal study of mathematics.

Instruction in Problem Solving

Problem solving is considered by some to be the centerpiece of mathematics reform (De Corte et al., 1996). Technology is increasingly replacing the need for hand calculations and accelerating the amount of data for analysis. Most importantly, a longstanding critique of traditional word problems by math educators has moved instruction toward complex problems (e.g., performance assessments) and sometimes highly authentic problem solving (e.g., anchored instruction).

In special education, there is limited research in problem solving, and it is often atheoretical in nature or guided largely by a behavioristic paradigm (Jitendra & Xin, 1997). Consequently, this kind of research has focused on teaching students with learning disabilities specific strategies for solving traditional word problems. These include having the students attend to key words in the problems (Darch, Carnine, & Gersten, 1984;

Gleason, Carnine, & Boriero, 1990; Wilson & Sindelar, 1991) as well as providing massed or “end of the chapter” practice around one kind of strategy for solving a highly structured set of problems (Carnine et al., 1997; Moore & Carnine, 1989). Mathematics educators have soundly criticized both approaches.

Metacognitive strategy training has been an exception in this pattern of special education research (e.g., Case, Harris, & Graham, 1992; Montague, 1995, 1997; Montague & Bos, 1986). Many math reformers regard metacognition to be the central feature of problem solving (DeCorte et al., 1996). Unfortunately, students with learning disabilities characteristically have a difficult time with metacognitive activities essential to effective problem solving. Their poor problem solving is also a result of difficulty in problem representations, and they tend to solve problems impulsively with little attention to the evaluation phase. Problem representation involves having students use the domain heuristic of graphically representing word problems using relational schematics (see Jitendra, 1999). There is a number of other domain heuristics (e.g., make a simpler version of the problem and solve it, look for a pattern, work backwards) that should be taught as well, particularly in the context of a wide range of math problems, not just word problems. These would seem to be reasonable extensions of previous metacognitive research in special education. Schoenfeld (1985) offers one model for successfully blending these domain heuristics with wider metacognitive strategies in his work on problem solving.

Recent work has focused on the role of guided instruction on complex or “anchored” problems (Bottge & Hasselbring, 1993; Goldman et al., 1997). Anchored instruction is another valuable direction in mathematics instruction because it is one of the few examples in the field that suggests how guided, classroom discussions work and how they can enhance higher order thinking. It is one of the few current efforts in mathematics

instruction for students with learning disabilities that draws on social constructivism in its attempt to address the “inert knowledge” problem (Bransford, Sherwood, Vye, & Rieser, 1986); that is, application of formal knowledge in highly situated contexts.

However, the growing support for anchored instruction as situated cognition (e.g., Gersten & Baker, 1998) should be tempered by some practical as well as theoretical considerations. For example, authentic problems are difficult to generate, and they are not necessarily as meaningful to students as they are to adults. Critics have raised similar concerns with complex, performance methods of assessment (Linn, Baker, & Dunbar, 1991). Recent work with middle school students with learning disabilities indicated that what curriculum developers initially thought were appropriate, real world problems had little relevance to the world of 12 and 13 year-olds (Woodward, 2001). In this case, researchers modified their problems to make them more authentic to middle school students, but their research also suggested that the problems still had to be balanced by more traditional exercises. Finally, the capacity of a teacher to include students with learning disabilities *adequately* in whole class discussions remains a problematic dimension of reform mathematics (for a detailed discussion of this problem, see Baxter, Woodward, & Olson, 2000).

De Corte et al. (1996) raised further concerns about highly situated instruction. They contended that highly situated problem solving does not necessarily generalize to the kinds of problem solving found in formal mathematics. For this reason, students with learning disabilities need continued opportunities to use general metacognitive as well as domain heuristic strategies on complex, but more contained problems. Under these conditions, the basic cognitive instructional principle of “less is more” applies. Instead of working through 10 to 15 one-step or “end of the chapter” word problems, students might

work through one or two problems completely in a lesson, and the classroom dialogue would include discussion of strategies (i.e., general metacognitive and domain specific heuristics), multiple solutions to problems, and a “debriefing” component that addressed what made these particular problems difficult or unique.

This shift toward an in-depth examination of one or two problems is not only likely to enhance the evaluation step in problem solving, but it will enable teachers to focus much more on classroom dialogue, particularly the role of what is loosely described in the literature as scaffolding. These elements of instruction in mathematical problem solving are under-described in the special education literature, and on the rare occasions where they occur, they stem from the behaviorist tradition. For example, Carnine and his colleagues (Carnine, Dixon, & Silbert, 1998; Kameenui & Carnine, 1998) present scaffolding as guided practice with step-by-step feedback that is eventually faded. On other accounts, scaffolding has been relegated to external devices such as cue cards for metacognition (see Stone, 1998).

A recent discussion of scaffolding demonstrates how poorly defined the construct of scaffolding has become. Several prominent special educators suggest that scaffolding has been reduced to a technique with little relation to theory (Reid, 1998; Stone, 1998; Wong, 1998). Stone (1993, 1998) also noted that research should focus on the linguistic or semiotic features of the student-teacher interaction as well as the way scaffolding is used on an individual basis in small group instruction. Scaffolding may be the wrong metaphor for teacher-student interactions in problem solving and that a broader method such as interactive instruction (Bos & Anders, 1990; Wong, Butler, Ficzere, & Kuperis, 1996) offers a better description of how teachers should foster problem solving in students with learning disabilities.

Regardless of how scaffolding or interactive instruction unfold in problem-solving research, there is a further need to articulate the "emotional" dimensions of problem-solving instruction. This is particularly true of late elementary and secondary students, and teachers need to be sensitive to the historical experiences of the learner. For students with learning disabilities, this often translates as the tendency to: 1) make negative attributions about their capacities as problem solvers specifically and, more generally, learners; 2) place little value on any number of mathematical activities; and 3) immediately elicit support from teachers in any number of ways once the mathematical problems become challenging. Again, there is very little in the special education or at-risk literature that articulates how teachers successfully mediate these challenges and move students toward the kind of mathematical problem solving that appears in the general education reform literature. In this regard, the work of attribution theorists (e.g., Covington, 1992; Weiner, 1992) is a helpful way of conceptualizing services for students with learning disabilities.

Concluding Remarks

Much of the special education intervention research that is consistent with the *NCTM Standards* and math reform is only at an emergent stage. However, the forces sustaining reform in this country typically yield an unambiguous message. We have moved from an era of hand computation practice to sense making in mathematics. While it is true that some special education researchers have investigated interventions consistent with math reform, others continue to focus on traditional topics in mathematics. What is missing is a synthesis of these approaches around topics that are commonly taught to students with learning disabilities. Even more the case, it is essential that this be done with an eye toward theory. The recent discussions around scaffolding (see Reid, 1998), for example, clearly

indicate how freely constructs are reduced to the level of technique with little regard for a guiding theoretical framework. Social constructivism, at least in its more contemporary form, provides such a framework for articulating skill development in the context of more complicated efforts.

This article provided a broad, early attempt at synthesizing where special education is with respect to mathematics reform. It is important to note that even though our discussion of mathematical topics was restricted to facts, computations, and problem solving, we do not imply that these should be the only topics of instruction for students with learning disabilities. A host of other important topics (e.g., geometry, probability, measurement) also warrant a significant place in a student's plan of instruction. In the end, a core concern in mathematics reform for students with learning disabilities is to go beyond the acquisition of knowledge of discrete topics such as the ones discussed in this article to a more comprehensive, integrated understanding of the discipline. Reformers (e.g., Cohen, McLaughlin, & Talbert, 1993; Resnick, 1987) long argued that mathematics, like other disciplines such as science or social studies, requires a particular "habit of mind" for sustained success as students move to higher and more complex levels of instruction in school. Arguably, this claim has increasing credibility as schools move to adopt reform curricula as well as technologies such as graphing calculators as part of middle school mathematics.

We also believe that a student's progress through these topics is likely to be uneven. For example, middle school students should be able to move relatively quickly through conceptually-guided computations en route to a more competent use of calculators, based in large part, on the nature of the instruction. On the other hand, facts, mental computation, and estimation will take much longer to develop, as the research on math fact acquisition

cited in this article suggests. This would also be true of competence in problem solving, where variations in the overall difficulty as well as specific semantic dimensions of problems will affect student success. Current work in this area (Woodward, Monroe, & Baxter, 2001) indicates that students can be strikingly independent in solving some multi-step or geometric problems after two months of instruction. Their success on other problems, however, indicates only a partial ability to use their strategic knowledge and a considerable level of interactive instruction is required. Successful, independent use of general metacognitive and domain specific heuristics for problem solving is likely to take years to develop in students with learning disabilities.

In closing, we concur with Hiebert (1999) who suggested that articulating the implications of mathematics reform for special education should be grounded in what we *value* educationally. This consideration is of paramount concern for students with learning disabilities who often have limited instructional time left before they complete their study of mathematics. Undoubtedly, there will be those in our field who will defend the “skills first” orientation to mathematics *in perpetuum*. However, reflection on what students with learning disabilities are likely to gain from this orientation suggests an isolated body of knowledge that has little application to the increasingly complex mathematics found at the middle and secondary school level and even less application to a world of work filled with computing devices. It is hoped that new directions in mathematics education will help to move students with learning disabilities out of a narrow and highly procedural set of experiences closer to the kind of mathematical instruction that is valued today.

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