Mathematics Education in the United States: Past to Present

John Woodward

University of Puget Sound

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Abstract

This article presents an historical review of mathematics education since the late 1950s in the United States. Three themes are used to organize the literature reviewed in the article: 1) broad sociopolitical forces, particularly highly publicized, educational policy statements; 2) trends in mathematics research, and 3) theories of learning and instruction. At times, these themes coincide, as was the case in the 1990s. In other cases such as the recent push for educational accountability, these themes conflict. Nonetheless, the themes go a long way to explain the serpentine nature of reform in the US over the last 45 years. This article also attempts to account for developments in special education as well as general education research, something which does not appear in most historical presentation of mathematics education.

There are many ways to describe the current state of mathematics education in the United States. It would be tempting simply to recount the success of math reform over the last 10 years. After all, the National Council of Teachers of Mathematics (NCTM) *Standards* have been the centerpiece of mathematics education in the US for more than a decade, and their influence has been apparent in the growth in reform-based research, curricula, and methods of assessment. Added to that, the *Standards* have moved all but one state in the country to adopt new content or curriculum standards in mathematics as of 2001.

Yet the actual nature of mathematics instruction in American classrooms, particularly special education settings, is more complicated. Highly traditional approaches exist alongside of instruction that matches the intentions of the NCTM *Standards* and associated reform efforts. There is also a resilient body of critics who, from time-to-time, cast doubt on systematic efforts to reform mathematics. In fact, the role basic skills advocates have played in the debate over best practice in American classrooms is one of the more interesting dimensions of what may best be described as a 50-year, serpentine path toward reform.

In order to elucidate the tensions in current mathematics education, as well as to provide a deeper context for an international audience, this article will offer an historical treatment of mathematics education in the US. There are a few excellent historical reviews (Kilpatrick, 1992; NCTM, 1970), but only rarely is special education research part of the presentation. One exception is Rivera's (1997) thoughtful discussion of general and special education research in mathematics.

This article will make explicit three themes in mathematics education since the 1950s: 1) broad sociopolitical forces, particularly highly publicized, educational policy statements; 2) trends in mathematics research, and 3) theories of learning and instruction. At times, these themes coincide, and there is somewhat uniform agreement about what needs to occur in American classrooms. On other occasions, there is conflict, leading to contradictory messages about the kind of mathematics that should be taught in American classrooms.

One final note is in order. The thematic structure of this article required a selective presentation of research. Unfortunately, many important contributions by general and special education math researchers were not included. The absence of their work in this article should not connote that their efforts are marginal to what we know about mathematics education today. On the contrary, their research, which may be either quantitative *or* qualitative in nature, has enriched our understanding of teaching and student learning in mathematics.

The 1950s and 1960s: Excellence in Education

The New Math

Kilpatrick (1992) labeled the 1950s and 1960s "the golden age" in the field of mathematics education because of the extensive federal funding for research and training in mathematics. A number of forces shaped this era, the most well known being the importance of mathematics to the development of atomic weapons in the 1940s and the Soviet's launch of Sputnik in 1957. America responded through a surge in federal funds to produce more scholars, teacher educators, and highly prepared mathematics teachers who would help the US compete internationally. Steelman's (1947) presidential report, *Manpower for Research*, articulated the need for successful secondary school mathematics programs that would eventually increase the number of engineers and other highly technically prepared workers needed for a more scientifically-oriented society. This report was indicative of the kind of high profile, policy documents that would appear over the next 50 years; ones exhorting education to produce more technically literate workers for an evolving economy.

Colleges and universities at the time were equally concerned about the low level of mathematics knowledge imparted by the K-12 education system. Many professors felt that entering students lacked an adequate computational and conceptual understanding of mathematics as well as the ability to apply this knowledge to other disciplines. Moreover, university educators were concerned about declining enrollments in their math courses. All of this suggested that the K-12 mathematics curriculum at the time was significantly out of date (Jones & Coxford, 1970; Kilpatrick, 1992). The general concern with low achievement in mathematics throughout the K-12 and collegiate systems, then, was one of the driving forces behind a push toward *excellence* in education (Lagemann, 2000).

One consequence of the excellence movement was a number of federally funded, university-based curriculum reform projects. Large scale curriculum development projects such as the University of Illinois Committee on School Mathematics, University of Illinois Arithmetic Project, and the School Mathematics Study Group were but a few examples of government funded reform efforts of the time.

The new math curricula of the late 1950s and 1960s are perhaps best remembered for their emphasis on instruction in abstract mathematical concepts at the elementary grades. These materials stressed topics such as set theory, operations and place value through different base systems (e.g., base 5 addition, base 3 subtraction), and alternative algorithms for division and operations on fractions. The "new math" was an attempt to introduce a formal understanding of mathematical principles and concepts from the early grades onward. Central ideas such as structure, proof, generalization, and abstraction were at the core of this reform (Jones & Coxford, 1970). A good example of the abstract nature of the mathematics can be found in several textbooks of the time (e.g., Rappaport, 1966; Riedesel, 1967), as shown in Figure 1 below.

<insert Figure 1 about here>

Mathematicians, like Max Beberman at the University of Illinois, became central to the new math movement. In his view, the goal of mathematics education was understanding and not simply the manipulation of symbols. Beberman stressed the importance of a precise vocabulary as well as the value of discovery learning. Given the right instructional materials, students should make observations and discover patterns, all of which would enhance transfer in learning (Lagemann, 2000).

The interest in discovery learning played out at a psychological level, where an array of educators were intent upon escaping the three decades of Thorndike's connectionist theory and, the more recent advent of Skinner's operant conditioning. As Rappaport (1966) lamented, behavioral approaches led to an atomization of knowledge where teachers controlled each step in the learning process. Behaviorism placed a premium on the efficient development of bonds through rote practice and memorization.

Rappaport (1966) and others favored gestalt psychology as a framework for helping students understand mathematics. Through gestalt principles, instruction could be organized around part-whole relationships and the continual reorganization of knowledge. Insight was also a key feature of learning, and this could be sponsored by activities that allowed students to analyze situations for patterns or structure. If a child were led to discover fundamental mathematical relationships, the kind of drill and practice advocated through connectionist psychology would be unnecessary. Moreover, students would be in a much better position to understand and explain "why" rather than merely tell "what."

Yet discovery learning was much more than simply putting children in front of manipulatives and having them figure out solutions. In this regard, the choice of the term "discovery" was an unfortunate choice of words. Riedesel (1967), for example, described an environment where teachers carefully guided learning in a way that is analogous to the current literature on scaffolded instruction. Once students were introduced to the problem, it was the teacher's job to move about the room and check for understanding. Students who were succeeding at the task were encouraged to make diagrams or write explanations that might help someone who was having difficulties with the problem. For those who were struggling, the teacher was to prompt thinking with mathematically relevant questions (e.g., "Could you use a diagram to solve this problem?"). There was considerable emphasis in Riedesel's vision of discovery learning on small group and classwide discussions. Moreover, it was apparent from his descriptions that teachers needed to possess a high level of pedagogical content knowledge. Not only did they have to understand the formal nature of the mathematics, but they needed to be able to ask "the right question at the right time." Grossnickle and Brueckner (1963), authors of a widely used text at the time, presented a similar account of mathematics lessons.

Equally influential at the time were Piaget's theories of child development as well as Bruner's early work in educational psychology. Although Bruner's main curricular work was in social studies, he used a number of mathematical examples in his seminal book, *Toward a Theory of Instruction* (1966), to help convey a vision of instruction where students explored contrasts, developed conjectures and hypotheses, and played structured games. His emphasis on carefully guided, "informed guessing" and conjecture also came to be associated with the concept of discovery learning. However, Bruner was careful in noting that this method would be far too time consuming given all of what a student needed to learn in a discipline like mathematics (Jones & Coxford, 1970). As Bruner (1966) commented, one of the key purposes of discovery learning was to give students confidence in their own ability to generate useful questions and hypotheses. Too much of instruction, particularly under the influence of behaviorism, had come to mean that students simply memorized what the teacher directed them to learn.

Slow Learners

Learning disabilities in mathematics was a nascent concept at this time. Definitions of learning disabilities in the 1960s acknowledged arithmetic disorders, but remediation was generally part of a set of older, perceptual motor techniques or emerging diagnostic-prescriptive approaches. These methods applied to a wide range of academic difficulties, not mathematics specifically. Once students were diagnosed by a clinician to determine a specific deficit or set of deficits, remediation techniques could range from visual-motor perceptual training to task analysis (Haring & Bateman, 1977).

Glennon and Wilson (1972) offered an unusually rich account of diagnostic-prescriptive methods in mathematics for this period of time, although it was applicable to the "slow learner" more than to students with learning disabilities per se. The authors are skeptical of Bruner's discovery methods because they do not help teachers select topics that should be taught to learners who, by definition, could not learn the same amount of content as the rest of the students in the classroom. They suggest that in addition to verbal and symbolic modes of instruction, teachers make use of manipulatives, pictorial representations, and conceivably, a student's cultural or ethnic background. In an effort to target instruction as specifically as possible, Glennon and Wilson draw upon Bloom's taxonomy (1956) and the work of Gagné (1970) as a way of identifying appropriate teaching methods and student outcomes. Finally, the authors are wary of instructional methods that simply repeat mechanical associations such as invert and multiply when learning how to divide fractions. They adopt the position that for slow learners, one needs to carefully target instructional objects, because the amount of material to be mastered over time will be less than that of more capable students.

The 1970s and 1980s: Equity and Excellence in Education

The new math of the 1960s foundered for a number of reasons, not the least of which was the abstract nature of the reform mathematics at the elementary school level. The lack of broad-based professional development for K-12 teachers also played a role in its demise. Teachers faced a situation where they needed to reconceptualize their understanding of mathematics. This resulted in many instances where the implementation of the new curricula failed (Moon, 1986). Another instrumental factor was the "back to basics" movement of the 1970s, which drove schools to place greater emphasis on reading, writing, and arithmetic. Educators re-envisioned teachers as the dominant figure in classroom instruction, and "frills" such as art, social services, and sex education were reconsidered as parts of the curriculum. Business leaders also complained that high school graduates were ill-prepared for employment, and many in the inner city complained that their children had been ignored and lacked basic skills (Brodinsky, 1977).

The National Institute of Education, which was established in 1972 to generate improved educational research and development, sponsored one of the more influential studies of mathematics education at the time. The Missouri Mathematics Effectiveness Project (Good, Grouws, & Ebmeier, 1983) began in the 1970s as an effort to describe the relationship between classroom teaching or processes and student achievement outcomes or products. This "process-product" research exemplified the efforts by educators to refine their understanding of basic skills instruction at the elementary school level, moving it from descriptive to experimental research. Good et al. underscored the importance of discerning critical teaching behaviors *within a subject matter* as they directly related to improved performance on standardized tests. The use of standardized tests as a core dependent measure became politically important to educational research during the 1970s and 1980s, as these tests became a significant index by which districts and states evaluated the quality of their schools. Good et al. acknowledged the significant limitations of standardized tests, but nonetheless felt that they were a reasonable and efficient way to measure student learning.

Good et al.'s (1983) active teaching model rendered a picture of effective math teachers who had good management skills, generally taught to the whole class, and kept a brisk pace throughout the period. Classroom questions were typically product-oriented (i.e., low level questions that yielded a correct answer), and offered process feedback (i.e., how to derive the answer) when students' answers were incorrect. Student feedback was generally immediate and non-evaluative. The active teaching model came to be associated with a highly formulaic style of teaching. Teachers began with a brief daily review (8 minutes), followed by the development portion of the lesson (20 minutes), independent seatwork (15 minutes), and a homework assignment. As one might suspect, the most variable and challenging component for teachers was in development portion of the lesson, where they were to focus on the meaning of the mathematics through lively demonstrations, process explanations, and controlled or guided practice. Those proficient at development could effectively explain why algorithms worked and how some skills were interrelated. It was thought that these methods would have been optimal for teaching a wide range of academic abilities in the classroom.

Federal educational policy and funded research projects also attended more to the significant disparities between the poor, who were largely African American, and children of higher socio-economic classes. The Johnson Administration's war on poverty was instrumental in creating initiatives that would demonstrate a significant commitment to *educational equity*. This commitment accelerated with federal legislation authorizing an array of services and legal protections for students with disabilities under the Educational of All Handicapped Children Act of 1975. If anything, this law characterized an American penchant for attending to individual differences in educational settings.

Large Scale Research in Basic Skills

In one of the largest federally funded studies of early education, Project Follow Through, attempted to determine the most effective methods for teaching basic skills to primary grade students in economically disadvantaged settings through a large quasi-experimental study. Mathematics was one of the core areas of instruction, and an analysis of competing models deemed the direct instruction method to be the most effective means of teaching disadvantaged students, at least in terms of performance on standardized tests like the Wide Range Achievement Test and the Metropolitan Achievement Test (Abt Associates, 1977). Even though these were exceedingly limited measures of mathematical competence, they were useful as a way of broadly measuring basic skills at the primary grade levels.

Direct instruction extended the active teaching model by adding a significant curricular component. Teachers and even instructional aides taught small groups using materials that prescribed exactly what teachers were to say. This was a significant instructional decision because it placed the burden of Good et al.'s (1983) development phase of the lesson on a static curriculum. Teachers and paraprofessionals had few liberties to expand on the content of a lesson because the program's daily scripts heavily prescribed what they were to do at each step.

The arithmetic instruction focused on a careful progression of basic skills that were taught in an explicit, step-by-step manner. Behavioral theory provided the framework for the mastery-oriented curriculum as well as its teaching practices (Becker, Engelmann, & Thomas, 1975a, 1975b). For example, first grade students were taught regrouping in addition through a carefully controlled progression of steps, beginning with small, non-regrouping problems and moving toward increasingly more complex ones. Prompts were used to signal students when and where to regroup. These prompts were eventually faded in subsequent lessons. Arguably, the nature of this arithmetic and the way it was taught was precisely the kind of "telling what" approach that many in the new math were rebelling against in the previous era.

Naturally, those who were intimately involved with the direct instruction model viewed the Follow Through evaluation findings as definitive evidence that the model provided a superior method for teaching subjects like arithmetic to low achieving students (Becker, 1977; Carnine, 2000; Gersten & Carnine, 1984; Gersten, Baker, Pugach, Scanlon, & Chard, 2001).

However, the entire study has long been subject to a range of criticisms, including the ad hoc way in which the study was designed and its choice of dependent measures (House, Glass, McLean, & Walker, 1978) to the confounding of key intervention variables (Lagemann, 2000). Cohen (1970) summarized the problem with the problem with studies like Follow Through as one where our decentralized educational structure overwhelmed the kind of controls necessary for successful, planned experimentation. More generally, math educators found these kinds of large scale statistical studies to be contradictory and deficient in the way they measured the thinking processes ostensibly being assessed by the dependent measures (Schoenfeld, 1987). Nonetheless, the Follow Through evaluation and its behavioral framework laid the foundation for subsequent research in mathematics involving students with learning disabilities.

Cognitive Science as a New Framework for Learning and Instruction

In contrast to the large scale, experimental research being conducted in the schools, an increasing number of educators and researchers turned to cognitive science as a framework for controlled mathematical investigations. Information processing theory was in its ascendancy during the 1970s and 1980s, and computers were appealing tools for modeling human thought processes. Cognitive scientists used computer simulations, indepth qualitative studies, and small scale experimental studies to investigate the processes behind mathematical understanding. Brown and Burton (1978) attempts at using artificial intelligence programs to examine student misconceptions in subtraction as well as Riley, Greeno, and Heller (1983) computer programs for analyzing and solving word problems were early efforts in the cognitive science movement in mathematics during this era.

By the 1980s, problem solving had become a central theme in mathematics education, and researchers like Silver (1987) hypothesized ways in which information processing theory could explain competent, if not expert problem solving. Of particular interest were the organization of information in long term memory, the role of visual images in enhancing understanding, and the importance of metacognition in the problem solving process.

Skemp's (1987) highly influential book, *The Psychology of Learning Mathematics,* underscored the importance of knowledge organization and the role of conceptual understanding in well developed schema. He expressed the same concern as math educators from the 1950s and 1960s about the role of behaviorism in classroom instruction and the extent to which students performed mathematics with little or no understanding. For this reason, Skemp emphasized reflective intelligence in mathematical understanding, or what was more generally called metacognition. Skemp's book was a reminder that the forces behind the new math movement not only had continued value in the math research community, but that the movement itself was international in scope (see Moon, 1986).

By the mid 1980s, cognitive research was the dominant framework in mathematics education. Cognitive scientists (Marr, 1982; Paivio, 1986) attempted to articulate the fundamental role of visual imagery as a representational form of memory and more specifically, as an aide to mathematical understanding (see Janvier, 1987). In addition to the continued work in metacognition and problem solving (e.g., Schoenfeld, 1985), researchers focused increasingly on the implication of information processing theory for teaching mathematic topics. Of specific interest was the relationship between conceptual and procedural understanding. For example, Hiebert and his colleagues (Hiebert, 1986; Hiebert & Behr, 1988) edited two influential books that described the importance of conceptual understanding in topics ranging from addition to decimals.

The late 1970s and 1980s was also the period when information processing theory was applied to the emergence of mathematical understanding in preschool children. Unlike behavioral accounts of learning, which focused almost exclusively on what was formally taught in schools, researchers (Baroody, 1987; Fuson, 1988; Gelman & Gallistel, 1978; Ginsburg, 1977; Siegler, 1978) documented the natural development of informal mathematical understanding in children. This body of work played a key role in reform efforts of the 1990s, where mathematics curriculum developers created materials for primary grade students that attempted to link an informal with a formal understanding of mathematics.

By the end of 1980s, a number of cognitively oriented math researchers were moving in the direction of constructivist theory. It is worth noting that the constructivist perspective was apparent in the research that was at the foundation of information processing theory. Anderson's

(1983) widely cited ACT theory not only called into question the central premises of behavioral theory, but presaged specific tenets of constructivism. "One of the fundamental assumptions of cognitive learning psychology is that new knowledge is in large part 'constructed' by the learner…. It means that mathematical knowledge – both the procedural knowledge of how to carry out mathematical manipulations and the conceptual knowledge of mathematical concepts and relationships – is always at least partly 'invented' by each individual learner" (Anderson, cited in Silver, 1987, p. 53).

The efforts by cognitive researchers to re-emphasize meaning and the role of conceptual understanding in mathematics coincided with broader educational policy statements published in the 1980s. *A Nation at Risk* (US Department of Education, 1983), one of the most important documents of the last quarter of the 20th century in the US, stressed *excellence* in education and, ironically, it was critical of the "back to basics" movement that was well under way in the country. Several important policy statements for mathematics education surfaced, beginning with *An Agenda for Action* (NCTM, 1980), and culminating in the *Curriculum and Evaluation Standards* (NCTM, 1989) and *Everybody Counts* (National Research Council, 1989). The sum effect of these policy statements was to re-invigorate the mathematic reform that re-emerged in the 1990s. However, for students with learning disabilities, reform efforts were slower to emerge.

<u>Mathematics Interventions for Students with Learning Disabilities</u>

Attention to mathematics instruction for students with learning disabilities in the 1970s and 1980s paled in comparison to the scope of research described immediately above. Moreover, mathematics was generally seen as a context for investigating learning disabilities and as a way of employing generalized interventions. Researchers were generally much more interested in the application of strategy instruction, direct instruction, or curriculum-based measurement as a generalized intervention framework for students with learning disabilities than a detailed analysis of mathematical topics.

Behaviorism persisted as a salient theoretical framework for research and as the basis for mathematics instruction. For example, Sugai and Smith (1986) used a multiple baseline design to validate a demonstration plus modeling technique for teaching subtraction to students with learning disabilities. Cybriwsky and Schuster (1990) used controlled time delay procedures (i.e., a specified period of time between the presentation of a multiplication fact and an instructional prompt) for teaching multiplication facts to an elementary aged student.

Building on the curricular work from Project Follow Through, direct instruction researchers conducted a number of studies involving one step word problems for basic operations (Darch, Carnine, & Gersten, 1984; Gleason, 1985; Gleason, Carnine, & Boriero, 1990; Wilson, & Sindelar, 1991). Students were taught a key word strategy that enabled them to discriminate multiplication and division from addition and subtraction word problems. Once students found key words such as *each* and *every,* they were taught to look for the “big number” as a way to next determine whether to divide or multiply.

Direct instruction curriculum developers also attempted to translate and extend their earlier work in arithmetic into different technology-based media. Videodisc technology was particularly appealing because it offered sufficient storage capacity that enabled a comprehensive approach to classroom instruction. Teachers could “play” entire lessons to a classroom of students using just one television monitor and videodisc player. Developers felt that this kind of technology would make classroom instruction more reliable because materials programmed into the videodisc would present what Good et al. (1983) described as the development portion of the lesson.

Research in fractions (Kelly, Gersten, & Carnine, 1990), and ratios (Moore & Carnine, 1989) employed contrasts between technology-based presentations of these topics and ones that represented traditional textbook approaches. In both instances, there was a heavy emphasis on algorithmic proficiency. Furthermore, the ratios study attempted to demonstrate the efficiency of a hierarchical, task analysis of fractions and ratios (see Carnine, Jitendra, & Silbert, 1997 for an extended discussion of this analysis). Problems as shown in Figure 2 illustrate how preskill instruction that teaches students to convert fractions to common denominators can be used to solve the problem once students identify key numbers and their associated terms.

<Insert Figure 2 about here>

As a counterpoint to the behaviorist's perspective, there was a growing interest in the information processing approaches. Swanson (1987) argued persuasively for information processing as a more comprehensive explanation of learning disabilities. Wong and her colleagues (Scruggs & Wong, 1990; Wong, 1982) also made significant contributions in the area of metacognition.

Much of the work in special education at this time focused on mental addition and the difficulties in learning math facts (Baroody, 1988; Geary, Widaman, Little, & Cormier, 1987; Goldman, Pellegrino, & Metz, 1988; Putnam, deBettencourt, and Leinhardt 1990; Russell & Ginsburg, 1984). One of the more ambitious intervention efforts was Hasselbring, Goin, and Bransford's (1988) program for teaching math facts at an automatic level. Given the importance of fluency in math facts as declarative knowledge in a wide range of mathematical tasks, their computer based program pretested students on known facts, built upon their existing knowledge of facts, taught facts in small instructional sets, used controlled response times to reinforce automaticity, and provided distributed practice on known facts as new facts were being taught.

An emphasis on strategy instruction in problem solving surfaced at the end of the 1980s. Goldman's (1989) comparative analysis of interventions for students with learning disabilities emphasized the importance of general and task specific strategies as part of instruction. Her essay was highly critical of behaviorist methods such as the ones cited above because they did little more than show students "what to do." Nor did these behavioral methods enhance transfer by helping students internalize meaningful strategies that facilitated problem representation. Instead, Goldman highlighted emerging work in self-instruction strategies and mediated performance. More indepth contributions of strategy instruction in word problem solving appeared in the 1990s.

The 1990s: Excellence in Education (Again)

The 1989 NCTM *Standards* coincided with a series of highly visible policy statements that, *in toto*, attempted to re-instill excellence in education. The *Standards* came at a time when the first Bush administration was looking for rigorous content area standards that would help push the United States to become first in the world in math and science. The Clinton administration changed the name of *America 2000* to *Goals 2000* and maintained an emphasis on high achievement in mathematics. The SCANS Report (US Department of Labor, 1991) as well as documents such as *America's Choice: High Skills or Low Wages!* (Commission on the Skills of the American Workforce, 1990) captured the country's anxiety about its transformation from a post-industrial to information economy.

These documents focused on the need for education to produce "knowledge workers" who were facile in the uses of technology, communication skills, and who possessed high levels of mathematical literacy. It was evident that computer technology was now reshaping the mathematics students needed to know now and in the future. At one end of the employment spectrum, bar code scanners and computerized cash registers reduced the need for many workers to actually calculate a customer's purchases or the change that was owed. For many workers in the 1990s, the spreadsheet became an instrumental tool for modeling complex mathematical problems. Perhaps one of the clearest insights into the rapid effects that technology was having on work -- and by extension, the literacy needed for work in the future -- came from a corporate executive who served on the SCANS commission. "What startles me about these descriptions [of the increasing number of jobs that depend on high levels of literacy] is the realization that they are accurate, but ten years ago I could not possibly have imagined them. What concerns me is this lack of imagination. What will our workplace look like ten years from today? " (US Department of Labor, 1991, p. 2). Five years later, an obscure technology that was later to be called *the internet* rapidly transformed global communication and commerce.

Another major factor that contributed to the importance of the 1989 NCTM *Standards* was the disenchantment with standardized tests as a gauge of student progress. As Linn (2000) noted in a comprehensive review of assessment trends over the last 20 years, there were multiple problems with the way standardized tests were administered and interpreted in the 1980s. Among the many criticisms of these measures was the way in which they tended to stress basic skills over complex understanding, a point other assessment experts echoed (e.g., Stiggins, 2001). Problems with standardized tests coupled with the call for more rigorous student outcomes led to the development in many states of performance based assessments and content area standards in mathematics that were taken directly from the 1989 NCTM *Standards.*

A related factor that heightened the importance of educational excellence in mathematics was the TIMSS research. Schmidt and his colleagues (Schmidt et al., 1996; Schmidt, McKnight, Cogan, Jakwerth, & Houang, 1999; Schmidt, McKnight, & Raizen, 1997) issued a series of reports following the 1995 international math and science assessments that were strongly critical of US mathematics practices. The US scored in the middle ranking of 41 countries, only significantly higher than countries like Cyprus and Lithuania. Asian countries and Belgium were significantly higher than the US in their overall performance.

A good deal has been written about Asian methods for teaching mathematics (see Stigler & Hiebert, 1999; Woodward & Ono, this issue). In a recent interview, Schmidt (Math Projects Journal, 2002) offered a broader assessment of effective mathematical practices based on curricular and pedagogical analyses of a range of high scoring countries. For Schmidt, a core theme that cut across the most successful countries is teachers who know mathematics and understand how to communicate concepts. Instruction is conceptual and not simply fragmented and algorithmic. The notion that US curricula tends to be "a mile wide and an inch deep" is the product of the TIMSS research.

Advances in mathematics research and education were more significant in the 1990s than at any point since the new math era decades earlier. One of the more striking contributions was the number of conceptual analyses of mathematical topics. These analyses covered elementary (Leinhardt, Putnam, & Hattrup, 1992), middle school (Carpenter, Fennema, & Romberg, 1993), and high school topics (Chazan, 2000). They also complemented the growing number of reform-based commercial curricula that were adopted by school districts through the US during this decade. By the end of the decade, a number of studies showed promise for reform based mathematics methods and curricula (Cohen & Hill, 2001; Fuson, Carroll, & Drueck, 2000; McCaffrey et al., 2001; Schoenfeld, 2002).

Mathematics research conducted within the US moved from a cognitive and information processing framework to a constructivist orientation. Drawing on philosophical, anthropological, and social psychological perspectives, constructivists attempted to provide an indepth picture of how learning occurs in context. Researchers drew on Vygotsky (1986) for insights into group dynamics and the role language played in learning. Lave (1988) and others (e.g., Brown, Collins, & Duguid, 1989; Cole, 1996) offered an anthropological perspective on the link between thought, culture, and the use of tools for communication and problem solving. Lave (1988) was especially articulate in her criticisms of traditional views of learning that promoted a "toolkit" view of knowledge. For Lave, the facts, procedures, and concepts taught in school math classes were not instruments that were merely pulled from a toolkit and used irrespective of context. Transfer was not that simple, and cultural context was a major factor in shaping our understanding of concepts. The degree to which culture inhibited transfer of learning spawned a number of interesting debates between information processing theorists and social constructivists (e.g., Anderson, Reder, & Simon, 1996; Hiebert et al., 1996, 1997).

Sociocultural perspectives became (and remain) the dominant framework for understanding mathematics learning and instruction. Making common mathematic topics (e.g., two digit addition problems, negative numbers, fractions) problematic and treating them as "ill-defined" were recurrent themes in sociocultural literature of the 1990s, all in an effort to get students to think mathematically (Ball, 1993; Carpenter, Fennema, Franke, Levi, & Empson, 1999; Cobb, Wood, & Yackel, 1993; Hiebert et al., 1997; Hiebert, Wearne, & Taber, 1991; Lampert, 1990; Lampert, Rittenhouse, & Crumbaugh, 1996). This perspective has influenced mathematics instruction for all students, including students with learning disabilities.

The Challenge to Teach *All* Students

From the beginning, a number of special educators were wary of the 1989 NCTM *Standards* and the ambitious call for more rigorous mathematics as well as the challenge to develop “mathematical power” in *all* students. There was virtually no mention in the *Standards* how the proposed curricular and pedagogical reforms would affect students with disabilities, the majority of whom were or would be receiving mathematics instruction in general education mathematics classes. At a practical level, both special and general educators were concerned about how these major shifts in curriculum and teaching methods would occur given the limited resources available to enact them. Consequently, the *Standards* were criticized as elitist (Hofmeister, 1993), too difficult to implement (Carnine et al., 1997), devoid of a research foundation (Carnine, 1992; Carnine, Jones, & Dixon, 1994; Kameenui, Chard, & Carnine, 1996; Stein & Carnine, 1999), and representative of discovery-oriented constructivism (Carnine et al., 1994; Mercer, Jordan, & Miller., 1994).

The response in the special education research community to the *Standards* and the reform mathematics of the 1990s was varied. For many special educators, the central issue involved systematic skills instruction and how (or if) traditional teaching practices could be aligned with reform mathematics. Even though behaviorism had waned as a useful psychological framework for mathematics instruction, there were occasional efforts to present its relevance to needs of students with learning disabilities.

For example, Carnine (1997) described a series of examples from the recently revised math curriculum used in the Follow Through Study from the 1970s. In addition to a more extensive treatment of the ratios example described earlier in this article, Carnine showed how the curriculum that he co-authored could reduce seven different formulas for volume (including a sphere) to variants of base *x* height formulas. He argued that students with learning disabilities could achieve a more robust understanding of the different volume formulas by focusing their attention on the base of each object. Unfortunately, he did not elaborate on how students would use this knowledge in a broader context or why the concept of base was more important than radius in understanding the volume of spheres. More interesting was Carnine's terse suggestion that this kind of instruction met the goals of the 1989 NCTM *Standards*. All of his other writings at the time were (and continue to be) harshly critical of the *Standards*.

In a more ambitious effort to make direct instruction relevant to research trends in the 1990s, Gersten and his colleagues (Gersten & Baker,1998; Baker, Gersten, & Lee, 2002) proposed a merger between behaviorism and social constructivism. The authors interpreted Bottge and Hasselbring's (1993) as an important example of how teachers could present highly algorithmic methods for performing basic operations on fractions for an extended period of time and then shift to constructivist methods for problem solving once students had mastered the algorithms.

There are a number of problems with these attempts to link behaviorism to contemporary mathematics, and this was apparent in other special education literature. First, the kind of hierarchical analysis that Carnine (1997) describes, one that leads to students solving complex ratio problems, can lead to a highly constrained level of understanding. Problems such as the ones presented in his article as well as in Figure 2 are what Goldman and her colleagues (Goldman Hasselbring, and the Cognition and Technology Group at Vanderbilt, 1997) call "end of the chapter" problems. They represent the kind of practice problems found in traditional math texts. For Goldman et al., curricular materials that are structured so that students answer highly predictable word problems that are based on associated practice in specific algorithms yields little in the way of conceptual understanding or the kind of flexibility needed to answer other types of problems. In the case of ratios, this instruction does little to communicate how ratios apply in geometry (e.g., the concept of pi, scaling figures) or how one would solve ratio problems that cannot be solved using the equivalent fractions algorithm (give an example) shown in Figure 2.

As for Gersten his colleagues (Gersten & Baker,1998; Baker et al., 2002), it is not clear at a theoretical or practical level how one merges two incompatible orientations (i.e., behaviorism and social constructivism). Behaviorism, particularly as it has been applied in special education, assumes a transmission view of knowledge. Through “explicit teaching,” an ambiguous term that the authors use to connote step-by-step highly directed instruction, the learner fully understands what the teacher is trying to communicate. This kind of explicit teaching tends to focus on mastery of declarative and procedural knowledge. Yet how well even these types of knowledge are retained over time is less a subject of research or discussion.

In contrast, cognitive and social constructivist views assume that interpretation by the learner and occasional misunderstandings are unavoidable. Direct transmission of complex concepts such as those found in a domain like mathematics is an illusory goal. Constructivists place a far greater emphasis on discussion, peer interactions, and conceptual understanding, all of which are at a great distance from what is found in the behavioral tradition of mathematics instruction in special education.

At stake here is what Cohen, McLaughlin, and Talbert (1993) called a “habit of mind.” How one learns a subject is a function of multiple factors, not the least of which are the ways a teacher orchestrates an environment for students and instructional materials. Teaching students for weeks to respond verbally to “right-wrong,” fast paced questions that are followed by workbook computation problems instills a habit of mind about learning mathematics. It is unlikely that teachers would or could suddenly change course after weeks of this kind of instruction and conduct lengthy, classwide discussions with students or have students grapple with ambiguous or seemingly contradictory concepts. At a practical level, the contrast in instructional methods is too great and too contradictory.

This problem is at the heart of Gersten and his colleagues' misinterpretation of Bottge and Hasselbring's (1993) fractions study. They confuse the design of the study (i.e., controlling for background knowledge by providing the same pre-intervention instruction in fractions to all students) with a prescription for pedagogy and instruction. In other words, Bottge and Hasselbring used a direct instruction videodisc program as part of their study in order to assure equivalence between the groups before the intervention portion of the study. This artifact of the study does not imply that mathematics should be taught this way, as confirmed by discussions with one of the co-authors of the study (T. Hasselbring, personal communication, January 28, 2000).

Furthermore, the anchored instruction literature (e.g., Goldman et al., 1997) suggests the opposite, particularly the notion that extended drill on algorithms may not lead to any kind of long term understanding. As noted earlier, this approach exemplifies the very problem Schmidt (Math Projects Journal, 2002) describes with mathematics instruction in the United States. Woodward, Baxter, and Robinson's (1999) research supports this point, as their findings indicate that when direct instruction curricula and methods are used to teach students with learning disabilities operations on decimals and percents, their retention of these algorithms degraded dramatically after the end of a four week intervention. It should be noted that these findings were omitted from Baker et al.'s (2002) synthesis of research on mathematics for low achieving students. Similar findings of poor performance over time as a result of direct instruction methods have been found in the area of subtraction (Woodward & Howard, 1994).

The work in anchored instruction during the 1990s best exemplified the influence of social constructivist movement as well as a clear counterpoint to the behaviorist influences that had pervaded special education for two decades. Bottge's research (Bottge, 1999, 2001; Bottge & Henrichs, Mehta, & Hung, 2002) has been a continuation of the efforts of the Cognition and Technology Group at Vanderbilt.

Anchored instruction researchers tend to answer the question of systematic skills practice by embedding relevant facts, procedures, and concepts in authentic problem solving situations. Students are immersed in complex environments, often accompanied by videodisc presentations, and they are required to sift through relevant and irrelevant information in order to formulate and solve problems (Goldman et al., 1997). Skills, then, are taught in context.

Yet anchored instruction is not without its problems. Linn, Baker, and Dunbar (1991) make a highly pertinent observation that was originally directed at the authentic assessment movement. What is considered meaningful and authentic to adults may not be the case for students. There is no reason to believe that all students will automatically engage in the highly contextualized problems developed by anchored instruction researchers. Math is still math, and the context for understanding and solving these problems take place in school classrooms. For some students, motivation remains a key factor.

A second problem with anchored instruction is that the precise nature of the skills and concepts learned in context are under-described in their literature. One typically reads extensive accounts of the problem solving environments, but precisely how and to what degree students learn relevant skills and concepts remains vague. This is a significant issue, particularly given the importance that the NCTM *Standards* place on conceptual understanding.

A final problem with anchored instruction can be found in the strategy instruction research conducted during the 1990s. This work carefully articulated the challenges that students with learning disabilities face in solving mathematics problem solving, albeit ones not as complex as those used in anchored instruction. Some of this work was part of a wider effort to teach students metacognitive strategies for academic tasks (e.g., Case, Harris, & Graham, 1992), while other studies can be directly linked to information processing theory research (Jitendra & Hoff, 1996; Jitendra, Hoff, & Beck, 1999) and the concern for metacognition in problem solving that occurred in mathematics research in the 1980s (e.g., Hutchinson, 1993).

Montague's research (Montague, 1992, 1995, 1997; Montague & Bos, 1986; Montague, Bos, & Doucette, 1991) in this area carefully attends to the difficulties that some students will face when they are asked to solve even common textbook problems. Montague draws on information processing and developmental theory to articulate the kind of delays that are indicative of late elementary and middle school students with learning disabilities. These students tend to have difficulty transforming linguistic and numerical information in word problems into appropriate mathematical equations and operations. They often resort to ineffective trial and error strategies as well as perform irrelevant computations. There is good reason to believe that students with learning disabilities would face comparable challenges in the kind of complex problems presented through anchored instruction.

Woodward and his colleagues (Baxter, Woodward, & Olson, 2001; Baxter, Woodward, Wong, & Voorhies, 2002; Woodward & Baxter, 1997) describe similar challenges that students with learning disabilities face in learning complex concepts and problems. Their naturalistic research in reform-oriented classrooms suggests that~~,~~ students with learning disabilities tend to exhibit difficulties with the cognitive load of the activities and curricular materials. Unless there is additional support in the class to mediate the instruction, these students tend to assume passive roles, and their progress is substantially below that of their non-learning disabled peers. These findings conflict with the earlier, anecdotal accounts from reform-based classrooms which suggested that at-risk students and ones with learning disabilities benefit from reform mathematics without special assistance (Fennema, Franke, Carpenter, & Carey, 1993; Resnick, Bill, Lesgold, & Leer, 1991).

It would appear that special education still needs a research-based model that articulates how skills and concepts are taught to students with learning disabilities in the context of reform based mathematics. After all, reform curricula require students to do more than solve either word problems or the more complex variety described by anchored instruction researchers. A good deal of conceptual understanding is necessary as well. For example, Smith's (1995) study of rational numbers presents a picture of high school students who are facile in comparing the size of fractions (e.g., 2/7 and 3/5) without resorting to common denominators. The kind of mental number sense that is evident here comes from representational activities and classroom discussions that develop a flexible understanding of concepts. Conceptual understanding is a significant part of mathematics reform that is only rarely described in the special education literature (see Baroody & Hume, 1991).

Contemporary Issues: Excellence and Accountability

One way to describe the current state of mathematics research and education in the United States is through the conflict between sustained reform and the broader politics of accountability. With the *Elementary and Secondary Act* of 2001 (a.k.a., the *No Child Left Behind Act*), the second Bush administration introduced the concept of "scientifically based research" as a mechanism for guiding instructional practices in classrooms throughout the country. This is an extraordinary shift in policy given the long history of decentralized educational decision making in this country. The political impact of *No Child Left Behind* extends beyond the states that must comply with the law. Even educators are being cajoled to reorganize their research community and follow scientifically based research principles (Feuer, Towne, & Shavelson, 2002). Although the *No Child Left Behind Act* was directed at reading education, there are indications that the Bush Administration will pursue scientifically based research in math. The administration has already quietly funded individuals who were instrumental in the back to basics movement in California during the 1990s (Hoff, 2002), and they are proposing scientifically based research into effective mathematics instruction through the newly organized Institute on Education Studies.

Questions about the research base of the NCTM *Standards* and, more generally, math reform are nothing new. Part of the concern has to do with the constructivist foundations of the reform. As Richardson (1997) observes, constructivism offers no one, simple perspective on teaching. The math reform literature contains examples of highly individual constructivism, social constructivism, and even emancipatory constructivism. What exactly constitutes constructivist practice remains ill defined in the minds of many, and it is particularly troublesome where teachers receive little or no professional development in these methods (Cohen & Hill, 2001).

Math researchers (e.g., Hiebert, 1999) also acknowledge that the *Standards* were never written as a research document and research cannot begin to account fully for all aspects of standards for a field. One central reason for this is that research support is *likely* to be uneven when standards are developed. Standards, after all, are policy documents. They are intended to promote further research in a given area as much as to reflect what is already known. Perhaps most importantly, standards reflect what is *valued* in education. Nonetheless, criticisms of the research underpinnings of the *Standards* and math reform have taken on a new force in the last two years.

In an invited article for the *Journal for Research in Mathematics Education*, Carnine and Gersten (2000) press for more rigorous designs to be used in mathematics research, particularly experimental and quasi-experimental techniques. They argue that more research of this type would possibly remedy what they perceive to be serious flaws in the *Standards*. Neither this article nor related writings (e.g., Gersten, Lloyd, & Baker, 2000) elaborate on why these methods are epistemologically superior to other techniques for doing research. What seems clearer is that the advocacy for experimental and quasi-experimental research is politically useful to organizations sympathetic of the view that mathematics should be focused on basic skills instruction. Large scale experimental research is a mechanism for questioning the validity of math reform research, so much of which has been based on qualitative methods.

For example, Carnine's work with members of the state board of education in California led to a review of 8,727 mathematics studies, only 110 of which were deemed to have had high quality designs (Dixon, Carnine, Lee, Wallin, & Chard, 1998). This report was highly influential in California's recent return to "the basics" in mathematics despite extensive criticism from AERA over the poor quality of the report and the limited inferences that could be made from the remaining 110 studies that met Carnine's acceptance criteria (see Becker & Jacob, 2000). In fairness to Carnine and his colleagues, the report made it clear in the listing of rejected studies that their own direct instruction research (e.g., Carnine's, Gersten's, Becker's) did not meet the criteria of high quality research either.

Gersten (2001) also used the experimental and quasi-experimental research criteria in a recent address to a US Department of Education conference on scientifically based research. He noted that there was little scientific research in mathematics and stated that the mathematics community was resistant to this kind of work, characterizing much of its research as romantic in nature.

To be sure, the tenor of these criticisms coincides with political views inside of the second Bush administration. It is ironic that math educators agree that quantitative research methods could yield a more balanced and reliable foundation for continued reform efforts (Burrill, 2001; Cohen, Raudenbush, & Ball, 2002; Research Advisory Committee, 2002). However, there are a number of important technical and theoretical considerations that must be taken into account.

First, research designs alone will not yield a satisfactory answer to the question, "what works?" *Scientific Research in Education* (National Research Council, 2002), which was commissioned by the National Research Council during the growing political debate over educational research and practice, makes this clear. This report attempts to reconcile the tensions between different types of research methods by noting that different questions require different methodologies. What makes quasi-experimental and experimental designs important isn't their epistemological superiority as much as their value to politicians and decision makers who fund education and educational research. Large scale experimental research has face validity to politicians because the methodology is similar to what is used in medical research. However, the extent to which educational research can ever resemble medical research is a highly controversial assumption (Feuer et al., 2002).

Perhaps most salient to the "high quality" standards mentioned above, the authors remind us that research is based on theory, and it is contingent upon values. Hiebert (1999) echoes this concern for values, arguing along with so many others in the math community that the kind of mathematics students need to know for the future is significantly different from that articulated in the basic skills era of the 1970s.

Second, there are major technical issues that must be addressed in large scale studies. Educational contexts play a significant role in this kind of research. Simply put, large scale research generates a considerable amount of variance, particularly within the implementation of a particular instructional approach. Too many potential interactions can be attributed to an untold number of factors. As Berliner (2002) reminds us, this was the problem with the Project Follow Through research. Controlling for contextual factors and the ensuing interactions would be an overwhelming problem for research teams, and it is likely to arise under the most unanticipated of circumstances. Clearly, it is much easier to prescribe high quality standards for research than it is to implement them in studies that occur over a significant period of time.

These technical and conceptual issues associated with large scale research become paramount in any investigation of contemporary mathematics methods. Cohen, Raudenbush, and Ball (2002) present a different kind of large scale experimental research design than the one that is commonly envisioned by social scientists. They argue that experimental studies which rely primarily on a narrow range of student outcome measures and simple models of instruction (e.g., teachers present information and students learn it) yield little useful information. Instructional environments are exceedingly complex. Ones that promote high levels of mathematical literacy effectively engage students in the learning process. Student motivation and attitudes toward mathematics are key variables. The pedagogical content knowledge that teachers have is a crucial resource, and it is not as easily quantified as years of teaching experience or highest degree obtained. Adjusting to student understanding is central to constructivist methods, and the fact that good teachers *calibrate* their instruction based on student needs makes detailed classroom observations and interviews imperative in a research agenda. All of this work occurs in a larger context where parents, principals, school policies, and state requirements mediate outcomes. These kinds of resources need to be taken into account for large scale research to be informative, which makes useful large scale research complex and extremely expensive.

Finally, the recent criticisms over the excessive number of qualitative research studies in mathematics education over the last 20 years need to be put in perspective. With little knowledge of mathematics research, one could incorrectly infer that these were intervention studies. Instead, part of the growth in mathematics research since 1980 has had to do with a wider scope of research than just intervention studies. Investigations of teacher thinking, pedagogical content knowledge, and student attitudes and beliefs have all been important research topics for mathematics educators (see Kelly & Lesh, 2000). In fact, one of the most influential research studies – one that is quoted by both sides of the skills-reform debate – was Ma's (1999) qualitative interviews with American and Chinese teachers. Her findings, along with policy research studies such as Cohen and Hill's (2001) examination of math reform in California, are important reminders of how teacher subject matter knowledge underpins what is or is not effective in the classroom. In fact, Cohen and Hill's extensive research of reform in California indicate that teachers who had adequate access to reform standards, materials, and professional development were a successful in teaching more complex mathematics to their students.

Learning Disabilities in Math

In addition to the broad concerns for reliable educational research, the second Bush administration has also re-examined the state of special education. The presidential commission policy document, *A New Era: Revitalizing Special Education* (US Department of Education, 2002), extends the theme of accountability found in the *No Child Left Behind Act* to special education practices in the US. Members of the presidential commission note that learning disabilities is one of the main problems with special education today. The number of students with learning disabilities has grown 300 percent since 1976, and there is an overrepresentation of minorities in this category.

The concern with the excessive number of students with LD can also be found in influential policy statements such as, *Rethinking Special Education for a New Century* (Finn, Rotherham, & Hokanson, 2001). In this report, Lyon et al. (2001) assert that far too many students with learning disabilities suffer from reading problems, and they would be better served by intense, early intervention programs. One implication of this view is that generally low achieving students who are now served in special education for subjects like reading and math would receive early interventions in other environments, presumably the regular classroom.

While Lyon et al. (2001) and his colleagues have put a great deal of effort into early interventions in reading which center around decoding instruction, this kind of work is less fully articulated in math. To be sure, the work of Geary (1994, this issue) and intervention programs like Right Start (Griffin, Case, & Siegler, 1994) offer important insights into the problems that academically low achieving students have with number sense as they enter schools. They are building blocks for future early intervention research.

However, one should be cautious about the long term effects that number sense interventions may yield. Academically low achieving students in the early grades face a number of other challenges (e.g., the ability to translate simple word problems into a computational form). Moreover, mathematics is a discipline where knowledge develops in a cumulative manner. As this historical review suggests, academically low achieving students – regardless of their status as students with learning disabilities – face any number of difficulties as they move from the early grades toward increasingly more complex topics.

How all of these federal efforts to insure that accountability accompanies excellence will unfold is far from certain at the time of this writing. What is clear to a number of educators is that there are good reasons why math reform needs to occur and why it is much more than an American phenomenon. After all, mathematics has been a central part of educational reform internationally since the late 1980s (Atkin & Black, 1997; De Corte, Greer, & Verschaffel, 1996), and this trend is apparent in many of the other articles in this special issue. Given the renewed interest in large scale experimental research, the question seems to be less, "what final answers will these kinds of studies provide?" and more, "how will this kind of research help us achieve a *convergent* understanding of best practices that is needed to move a discipline like mathematics forward?"

References

Abt Associates (1977). Education as experimentation: A planned variation model. Cambridge, MA: Abt Associates.

Anderson, J. (1983). The architecture of cognition. Cambridge, MA: Harvard University Press.

Anderson, J., Reder, L.., & Simon, H. (1996). Situated learning and education. Educational Researcher, 25(4), 5-11.

Atkin, M., & Black, P. (1997). Policy perils of international comparison. Phi Delta Kappan, 79(1), 22-28.

Baker, S., Gersten, R., Lee, D. (2002). A synthesis of empirical research on teaching mathematics to low-acheiving students. Elementary School Journal, 103(2), 45-74.

Baroody, A. (1987). Children's mathematical thinking: A developmental framework for preschool, primary, and special education teachers. New York: Teachers College Press.

Baroody, A. (1988). Mental addition development of children classified as mentally handicapped. Educational Studies in Mathematics, 19, 369-388.

Baroody, A., & Hume, J. (1991). Meaningful mathematics instruction: The case of fractions. Remedial and Special Education, 12(3), 54-68.

Baxter, J., & Woodward, J., & Olson, D. (2001). Effects of reform-based mathematics instruction in five third grade classrooms. Elementary School Journal, 101(5), 529-548.

Baxter, J., Woodward, J., Wong, J., and Voorhies, J. (2002). We talk about it, but do they get it? Learning Disabilities Research and Practice, 17(3), 173-185.

Becker, W. (1977). Teaching reading and language to the disadvantaged – What we have learned from field research. Harvard Educational Review, 47 (4), 518-542.

Becker, W., Engelmann, S., & Thomas, D. (1975a). Teaching 1: Classroom management. Palo Alto, CA: Science Research Associates.

Becker, W., Engelmann, S., & Thomas, D. (1975a). Teaching 2: Cognitive learning and instruction. Palo Alto, CA: Science Research Associates.

Becker, J., & Jacob, B. (2000). The politics of California school mathematics: The anti-reform of 1997-99. Phi Delta Kappan, 81(7), 529-537.

Berliner, D. (2002). Educational research: The hardest science of all. Educational Researcher, 31(8), 18-20.

Bloom, B. (1956). Taxonomy of educational objectives~~, the~~: The classification of educational goals. New York: Longmans, Green, & Co.

Bottge, B. (1999). Effects of contextualized math instruction on problem solving of average and below-average achieving students. Journal of Special Education, 33, 81-92.

Bottge, B. (2001). Reconceptualizing math problem solving for low-achieving students. Remedial and Special Education, 22, 102-112.

Bottge, B., & Hasselbring, T. (1993). A comparison of two approaches for teaching complex, authentic mathematics problems to adolescents in remedial math classes. Exceptional Children, 59(6), 545-556.

Bottge, B., & Henrichs, M., Mehta, Z., & Hung, Y. (2002). Weighing the benefits of anchored math instruction for students with disabilities in general education classes. Remedial and Special Education, 22, 299-314.

Brodinsky, B. (1977). Back to the basics: The movement and its meaning. Phi Delta Kappan, 58(7), 522-527.

Brown, J. & Burton, R. (1978). Diagnostic models for procedural bugs in basic mathematic skills. Cognitive Science, 2, 155-168.

Brown, J.S., Collins, A., & Duguid, P. (1989). Situated cognition and the culture of learning. Educational Researcher, 18(1), 32-41.

Bruner, J. (1966). Toward a theory of instruction. New York: W.W. Norton & Company.

Burrill, G. (2001). Mathematics education: The future and the past create a context for today's issues. In T. Loveless (Ed.), The great curriculum debate: How should we teach reading and math? (pp. 25-41). Washington, D.C.: Brookings Insitution Press.

Carnine, D., A. Jitendra, & Silbert, J. (1997). A descriptive analysis of mathematics curricular materials from a pedagogical perspective. Remedial and Special Education, 18(2): 66-81.

Carnine, D. (1992). Expanding the notion of teachers' rights: Access to tools that work. Journal of Applied Behavior Analysis, 25(1), 7-13.

Carnine, D. (1997). Instructional design in mathematics for students with learning disabilities. Journal of Learning Disabilities, 30(2), 130-141.

Carnine, D. (2000). Why education experts resist effective practices (and what it would take to make education more like medicine). Washington, DC: Fordham Foundation. Retrieved November 15, 2002, from http://www.edexcellence.net/library/carnine.html

Carnine, D., & Gersten, R. (2000). The nature and roles of research in improving achievement in mathematics. Journal for Research in Mathematics Education, 31(2), 138-143.

Carnine, D., Jones, E., & Dixon, R. (1994). Mathematics: Educational tools for diverse learners. School Psychology Review, 23(3), 406-427.

Carpenter, T., Fennema, E., & Romberg, T. (1993). Rational numbers: An integration of research. Hillsdale, NJ: Erlbaum.

Carpenter, T., Fennema, E., Franke, M., Levi, L., & Empson, S. (1999). Children's mathematics: Cognitively guided instruction. Portsmouth, NH: Heinemann.

Case, P., Harris, K., & Graham, S. (1992). Improving the mathematical problem-solving skills of students with learning disabilities: Self-regulated strategy development. The Journal of Special Education, 26(1), 1-19.

Chazan, D. (2000). Beyond formulas in mathematics teaching: Dynamics of the high school algebra classroom. New York: Teachers College Press.

Cobb, P., Wood, T., & Yackel, E. (1993). Discourse, mathematical thinking, and classroom practice. In E. Forman, N. Minick, & C. Stone (Eds.), Contexts for learning: Sociocultural dynamics in children's development (pp. 91-119). New York: Oxford University Press.

Cohen, D. (1970). Politics and research: Evaluation of social action programs in education. Review of Educational Research, 40(3), 213-328.

Cohen, D. & Hill, H. (2001). Learning policy: When state education reform works. New Haven, CT: Yale University Press.

Cohen, D., Raudenbush, S., Ball, D. (2002). Resources, instruction, and research. In F. Mosteller & R. Boruch (Eds.), Evidence matters: Randomized trials in education research (pp. 80-119). Washington, D.C.: Brookings Institution Press.

Cole, M. (1996). Cultural psychology: A once and future discipline. Cambridge, MA: Harvard University Press.

Commission on the Skills of the American Workforce (1990). America's choice: High skills or low wages! Rochester, N.Y.: National Center on Education and the Economy.

Darch, C., Carnine, D., & Gersten, R. (1984). Explicit instruction in mathematics problem solving. Journal of Educational Research, 77(66), 350-359.

De Corte, E., Greer, B., & Verschaffel, L. (1996). Mathematics teaching and learning. In D. Berliner & R. Calfee (Eds.), Handbook of Educational Psychology (pp. 491-549). New York: Macmillan.

DeVault, M., & Weaver, J. (1970). Forces and issues related to curriculum and instruction, K-6. In NCTM (Ed.), A history of mathematics education in the United States and Canada (pp. 93-154). Washington, DC: National Council of Teachers of Mathematics.

Dixon, R., Carnine, D., Lee, D., Wallin, J., & Chard, D. (1998). Review of high quality experimental mathematics research: Report to the California State Board of Education, Eugene, OR: National Center to Improve the Tools of Educators.

Fennema, E., Franke, M., Carpenter, T., & Carey, D. (1993). Using children's mathematical knowledge in instruction. American Educational Research Journal, 30(3), 555-583.

Finn, Rotherham, & Hokanson (2001). Rethinking special education for a new century. Washington, D.C.: Fordham Foundation.

Feuer, M., Towne, L;, & Shavelson, R. (2002). Scientific culture and educational research. Educational Researcher, 31(8), 4-14.

Fuson, K. (1988). Children's counting and concepts of number. New York: Springer-Verlag.

Fuson, K., Carroll, W., Drueck, J. (2000). Achievement results for second and third graders using *standards*-based curriculum *everyday mathematics*. Journal for Research in Mathematics Education, 31(3), 277-295.

Gagne, R. (1970). The conditions of learning (2nd edition). New York: Holt, Rinehardt, & Winston.

Geary, D. (1994). Children's mathematical development. Washington, DC: American Psychological Association.

Geary, D., Widaman, K., & Cormier, P. (1987). Cognitive addition: Comparison of learning disabled and academically normal elementary school children. Cognitive Development, 2, 249-269.

Gelman, R. & Gallistel, C. (1978). The child's understanding of number. Cambridge, MA : Harvard University Press.

Gersten, R. (2001). Mathematics education and achievement. US Department of Education Conference on Scientifically Based Research. Retrieved October 3, 2002, from http://www.ed.gov/nclb/research/gersten.html

Gersten, R., & Baker, S. (1998). Real world use of scientific concepts: Integrating situated cognition with explicit instruction. Exceptional Children, 65(1), 23-36.

Gersten, R., Baker, S., Pugach, M., Scanlon, D., & Chard, D. (2001). Contemporary research in special education. In V. Richardson (Ed.) Handbook of research on teaching (Fourth Edition). (pp. 695-722). Washington, DC: American Educational Research Association.

Gersten, R., Lloyd, J., & Baker, S. (2000). Designing high quality research in special education: Group design experiments. Journal of Special Education, 24(1), 218-237.

Ginsburg, H. (1977). Children's arithmetic: The learning process. New York: D. Van Nostrand Co.

Gleason, M. (1985) A comparison of two CAI approaches in teaching math word problems to handicapped learners. University of Oregon.

Gleason, M., Carnine, D., & Boriero, D. (1990). Improving CAI effectiveness with attention to instructional design in teaching story problems to mildly handicapped students. Journal of Special Education Technology, 10(3), 129-136.

Goldman, S. (1989). Strategy instruction in mathematics. Learning Disabilities Quarterly, 12, 43-55.

Goldman, S., Hasselbring, & the Cognition and Technology Group at Vanderbilt (1997). Achieving meaningful mathematics literacy for students with learning disabilities. Journal of Learning Disabilities, 30(2), 198-208.

Goldman, S. R., Pellegrino, J. W., & Mertz, D. L. (1988). Extended practice of basic addition facts: Strategy changes in learning disabled students. Cognition and Instruction, 5, 223-265.

Griffin, S., Case, R., & Siegler, R. (1994). Rightstart: Providing the central conceptual prerequisites for first formal learning of arithmetic to students at risk for school failure. In K. McGilly (Ed.), Classroom lessons: Integrating cognitive theory and classroom practice (pp. 25-49), Cambridge, MA: MIT Press.

Grossnickle, F., Brueckner, L. (1963). Discovering meaning in elementary school mathematics (4th edition). New York: Holt, Rinehart, and Winston.

Haring, N., & Bateman, B. (1977). Teaching the learning disabled child. Engelwood Cliffs, NJ: Prentice-Hall, Inc.

Hasselbring, T. S., Goin, L. I., & Bransford, J. D. (1988). Developing math automaticity in learning handicapped children: The role of computerized drill and practice. Focus on Exceptional Children, 20(6), 1-7.

Hiebert, J. (1986). Conceptual and procedural knowledge: The case of mathematics. Hillsdale, NJ: Erlbaum.

Hiebert, J. (1999). Relationships between research and the NCTM Standards. Journal of Research in Mathematics Education, 30(1), 3-19.

Hiebert, J., & Behr, M. (1988). Number concepts and operations in the middle grades. Hillsdale, NJ: LEA.

Hiebert, J., Carpenter, T., Fennema, E., Fuson, K., Wearne, D., Murray, H., Olivier, A., & Human, P. (1996). Problem solving as a basis for reform in curriculum and instruction: The case of mathematics. Educational Researcher, 25(4), 12-21.

Hiebert, J., Carpenter, T., Fennema, E., Fuson, K., Wearne, D., Murray, H., Olivier, A., & Human, P. (1997). Making sense: Teaching and learning mathematics with understanding. Portsmouth, NH: Heinemann.

Hiebert, J., Wearne, D., & Taber, S. (1991). Fourth graders' gradual construction of decimal fractions during instruction using different physical representations. Elementary School Journal, 91, 321-41.

Hoff, D. (2002). Bush to push for math and science upgrade. Education Week. 22(12), 19,24.

Hofmeister, A. (1993). Elitism and reform in school mathematics. Remedial and Special Education 14 (6), 8-13.

House, E., Glass, G., McLean, L. & Walker, D. (1978). No simple answer: Critique of the follow through evaluation. Harvard Education Review, 48(2), 128-160.

Hutchinson, N. (1993). Effects of cognitive strategy instruction on algebra problem solving of adolescents with learning disabilities. Learning Disabilities Quarterly, 16, 34-63.

Janvier, C. (1987). Problems of representation in the teaching and learning of mathematics. Hillsdale, NJ: Erlbaum.

Jitendra, A., & Hoff, K. (1996). The effects of schema-based instruction on mathematical word problem solving performance of students with learning disabilities. Journal of Learning Disabilities, 29, 422-431.

Jitendra, A., Hoff, K., & Beck, M. (1999). Teaching middle school students with learning disabilities to solve word problems using a schema-based approach. Remedial and Special Education, 20(1), 50-64.

Jones, P., & Coxford, A. (1970). Mathematics in the evolving schools. In NCTM (Ed.), A history of mathematics education in the United States and Canada (pp. 11-92). Washington, DC: National Council of Teachers of Mathematics.

Kameenui, E., Chard, D., Carnine, D. (1996). Response: The new school mathematics and the age-old dilemma of diversity: Cutting or untying the gordian knot. In M. Pugach & C. Warger (Eds.), Curriculum trends, special education, and reform (pp. 94-106). New York: Teachers College Press.

Kelly, A., & Lesh, R. (2000). Handbook of research design in mathematics and science education. Mahwah, N.J.: Erlbaum.

Kelly, B., Gersten, R., & Carnine, D. (1990). Student error patterns as a function of curriculum design: Teaching fractions to remedial high school students and high school students with learning disabilities. Journal of Learning Disabilities, 23, 23-29.

Kilpatrick, J. (1992). A history of research in mathematics education. In D. Grouws (Ed.), Handbook of research on mathematics research and teaching. (pp. 3-39). New York: MacMillan.

Lagemann, E. (2000). An elusive science: The troubling history of education research. Chicago: The University of Chicago Press.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. American Educational Research Journal, 27(1), 29-63.

Lampert, M., Rittenhouse, P., & Crumbaugh, C. (1996). Agreeing to disagree: Developing sociable mathematics discourse. In D. Olson & N. Torrance (Eds.), The handbook of education and human development (pp. 731-764). Cambridge, MA: Blackwell Publishers Ltd.

Lave, J. (1988). Cognition in practice. Cambridge: Cambridge University Press.

Leinhardt, G., Putnam, R., & Hattrup, R. (1992). Analysis of arithmetic for mathematics teaching. Hillsdale, NJ: Erlbaum.

Linn, R. (2000). Assessments and accountability. Educational Researcher, 29(2), 4-16.

Lyon, G. R., Fletcher, J., Shaywitz, S., Shaywitz, B., Torgesen, J., Wood, F., Schulte, A., & Olson, R. (2001). Rethinking learning disabilities. In C. Finn, A. Rotherham, & C. Hokanson (Eds.), Rethinking special education for a new century (pp.259-288). Washington, D.C.: Fordham Foundation.

McCaffrey, D., Hamilton, L;, Stecher, B., Klein, S., Bugliari, D., & Robyn, A. (2001). Interactions among instructional practices, curriculum, and student achievement: The case of standards-based high school mathematics. Journal for Research in Mathematics Education, 32(5), 493-517.

Ma, L. (1999). Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States. Mahwah, N.J.: Erlbaum.

Marr, D. (1982). Vision : a computational investigation into the human representation and processing of visual information. San Francisco: Freeman.

Math Projects Journal. (2002). TIMSS: A call for substance – An interview with Dr. William Schmidt. Retrieved on December 5, 2002, from http://www.mathprojects.com/scores/timss.asp#Link1.

Mercer, C., Jordan, L., & Miller, S. (1994). Implications of constructivism for teaching math to students with moderate to mild disabilities. The Journal of Special Education, 28(3), 290-306.

Montague, M. (1992). The effects of cognitive and metacognitive strategy instruction on the mathematical problem solving of middle school students with learning disabilities. Journal of Learning Disabilities, 25(4), 230-248.

Montague, M. (1995). Cognitive instruction and mathematics: Implications for students with learning disorders. Focus on learning problems in mathematics, 17(2), 39-49.

Montague, M. (1997). Cognitive strategy instruction in mathematics for students with learning disabilities. Journal of Learning Disabilities, 30, 164-177.

Montague, M., & Bos, C. (1986). Cognitive strategy training on verbal math problem solving performance of learning disabled adolescents. Journal of Learning Disabilities, 19, 26-33.

Montague, M., Bos, C., & Doucette, M. (1991). Affective, cognitive, and metacognitive attributes of eighth-grade mathematical problem solvers. Learning Disabilities Research & Practice, 6, 145-151.

Moon, B. (1986). The ‘new maths’ curriculum controversy: An international story. Philadephia, PA: The Falmer Press.

Moore, L., & Carnine, D. (1989). A comparison of two approaches to teaching ratios and proportions to remedial and learning disabled students: Active teaching with either basal or empirically validated curriculum design material. Remedial and Special Education, 10(4), 28-37.

National Council of Teachers of Mathematics (1970). A history of mathematics education in the United States and Canada. Washington, DC: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (1980). An agenda for action. Reston, VA: NCTM.

National Research Council. (2002). Scientific research in education. R. Shavelson & L. Towne (Eds.), Committee on Scientific Principles for Educational Research. Washington, D.C.: National Academy Press.

Paivio, A. (1986). Mental representations: A dual coding approach. New York: Oxford.

Putnam, R., deBettencourt, L., & Leinhardt, G. (1990). Understanding of derived-fact strategies in addition and subtraction. Cognition and Instruction, 7(3), 245-285.

Rappaport, D. (1966). Understanding and teaching elementary school mathematics. New York: John Wiley & Sons, Inc.

Research Advisory Committee (2002). Mathematics education research: Can the field deliver? Journal for Research in Mathematics Education, 33(5), 313-318.

Resnick, L., Bill, V., Lesgold, S., & Leer, M. (1991). Thinking in arithmetic class. In B. Means, C. Chelemer, & M. Knapp (Eds.), Teaching advanced skills to at-risk students (pp. 27-53). San Francisco: Jossey-Bass.

Riedesel, C. (1967). Guiding discovery in elementary school mathematics. New York: Appleton-Century-Crofts.

Riley, M., Greeno, J., & Heller, J. (1983). Development of children's problem solving ability in arithmetic. In H. Ginsburg (Ed.). Development of mathematical thinking (pp. 134-157). New York: Academic Press.

Rivera, D. (1997). Mathematics education and students with learning disabilities: Introduction to the special series. Journal of Learning Disabilities, 30(1), 2-19.

Russell, R., & Ginsburg, H. (1984). Cognitive analysis of children's mathematics difficulties. Cognition and Instruction, 1, 217-244.

Schmidt, W., Jorde, D., Cogan, L., Barrier, E., Gonzalo, I., Moser, U., & Shimizu, K. (1996). Characterizing pedagogical flow: An investigation of mathematics and science teaching in six countries. Boston: Kluwer Academic Publishers.

Schmidt, W., McKnight, C., Cogan, L., Jakwerth, P., & Houang, R. (1999). Facing the consequences: Using TIMSS for a closer look at U.S. mathematics and science education. Dordrecht: Academic Publishers.

Schmidt, W., McKnight, C., & Raizen, S. (1997). A splintered vision: An investigation of U.S. science and mathematics education. Dordrecht: Academic Publishers.

Schoenfeld, A. (1985). Mathematical problem solving. Orlando, FL: Academic Press.

Schoenfeld, A. (1987). Cognitive science and mathematics education overview. In A. Schoenfeld (Ed). Cognitive science and mathematics education (pp.1-32). Hillsdale, NJ: LEA.

Schoenfeld, A. (2002). Making mathematics work for all children: Issues of standards, testing, and equity. Educational Researcher, 31(1), 13-25.

Scruggs, T., & Wong, B. (1990). Intervention research in learning disabilities. New York: Springer.

Siegler, R. (1978). Children's thinking : What develops? Hillsdale, NJ: LEA.

Silver, E. (1987). Cognitive theory and research. In A. Schoenfeld (Ed.), Cognitive science and mathematics education (pp.33-60). Hillsdale, NJ: LEA.

Skemp, R. (1987). The psychology of learning mathematics. Hillsdale, NJ: LEA.

Smith, J. (1995). Competent reasoning with rational numbers. Cognition and Instruction, 13(1), 3-50.

Steelman, J. (1947). Manpower for research. Washington, DC: Government Printing Office.

Stein, M., & Carnine, D. (1999). Designing and delivering effective mathematics instruction. In R. Stevens (Ed.), Teaching in American schools (pp. 245-270). Upper Saddle River, NJ: Merrill.

Stiggins, R. (2001). Student-centered classroom assessment (3rd Edition). Englewood Cliffs, NJ: Prentice Hall.

Stigler, J., & Hiebert, J. (1999). The teaching gap. New York: Simon & Schuster.

Swanson, H. (1987). Information processing theory and learning disabilities: An overview. Journal of Learning Disabilities, 20, 3-7.

U.S. Department of Labor. (1991). What work requires of schools: A SCANS report for America 2000. Washington, D.C.: Secretary's Commission on Achieving Necessary Skills, U.S. Dept. of Labor.

U.S. Department of Education (2002). A New Era: Revitalizing Special Education. Jessup, MD: Education Publications Center.

Vygotsky, L. (1986). Thought and language. Cambridge, MA: MIT Press

Wong, B. (1982). Metacognition and learning disabilities. Gaithersburg, Md.: Aspen Systems Corp.

Woodward, J., Baxter, J., & Robinson, R. (1999). Rules and reasons: Decimal instruction for academically low achieving students. Learning Disabilities Research & Practice, 14(1), 15-24.

Woodward, J., & Baxter, J. (1997). The effects of an innovative approach to mathematics on academically low achieving students in mainstreamed settings. Exceptional Children, 63(3), 373-388.

Woodward, J., & Howard, L. (1994). The misconceptions of youth: Errors and their mathematical meaning. Exceptional Children, 61(2), 126-136.

Figure 1

New Math and Fractions

$$\frac{a}{b} + \frac{c}{d} = \frac{(a \times d) + (b \times c)}{b \times d} \quad \text{or} \quad \frac{ad + bc}{bd}$$

Figure 2

Ratios Word Problem

Workers in a factory can assemble 3 motorcycles in 2 hours. How many hours would it take to assemble 21 motorcycles?

$$\frac{\text{motorcycles}}{\text{hours}} \qquad \frac{3}{2} \quad \mathbf{x} \quad \frac{7}{7} \quad = \quad \frac{21}{\boxed{}}$$