The Effects of an Innovative Approach to Mathematics on Academically Low Achieving Students in Mainstreamed Settings

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Abstract

This article presents results from a year-long study of an innovative approach to mathematics and its impact on students with learning disabilities as well as those at-risk for special education. There is a considerable interest in the field regarding current mathematics reform, particularly as it reflects the simultaneous and conflicting movements toward national standards and inclusion. Results suggest that innovative methods in mathematics are viable for students with average and above average academic abilities and that students with learning disabilities or those at-risk for special education need much greater assistance if they are to be included in general education classrooms. The success of the majority of students in this study raises questions about commonly advocated methods in special education.

Introduction

The National Council of Teachers of Mathematics *Curriculum and Evaluation Standards* (NCTM, 1989) reflect a high level of consensus within the mathematics education community about current and future directions of the discipline. The *Standards* are intended as a policy document for professionals in mathematics education as well as a vision of excellence, one which attempts to move the field well beyond the minimal competencies of the back-to-basic movement of the 1980s (Bishop, 1990).

While the *Standards* are the most visible component of math reform for many, particularly special education researchers, it should be noted that they reflect almost two decades of research, curriculum development, and related policy documents by the NCTM and other professional organizations. The research, which draws extensively on cognitive psychology and child development (e.g., Gelman & Gallistel, 1978; Grouws, 1992; Hiebert, 1986; Putnam, Lampert, & Peterson, 1990), is a considerable enhancement of the knowledge base which led to the "New Math" movement of the early 1960s.

Mathematics education research over the last ten years has also yielded detailed analyses of elementary and secondary math concepts (Carpenter, Fennema, & Romberg, 1993; Hiebert & Behr, 1988; Leinhardt, Putnam, & Hattrup, 1992). More recently, a series of research-based curricula have emerged (e.g., *Everyday Mathematics*, Bell, Bell, & Hartfield, 1993). Finally, policy documents such as *An Agenda for Action* (NCTM, 1980) and Everybody *Counts* (National Research Council, 1989) consistently argued for significant changes in the role of computational practice and the type of problem solving found in most commercial textbooks, as well as an increased role for technology.

Despite the depth of the reform in mathematics, special educators have most of their concern over the potential impact of the NCTM *Standards,* which they feel reflect a wider, national standards movement. There is little mention in the *Standards*, or for that matter, *Goals 2000,* regarding the role of students with disabilities or how their unique needs will be addressed. For example, the *Standards* press for higher student performance through more challenging curriculum: specifically, a greater emphasis on conceptual understanding and having students solve longer, less well-defined problems. Pushing *all* students to achieve higher academic goals would seem to directly clash with the move to include more and more special education students in general education classrooms where little if any additional support is provided (Carnine, Jones, & Dixon, 1994; Fuchs & Fuchs, 1994). After all, problems accommodating students with learning disabilities in *traditional*, general education classrooms are well documented in recent case study research (Baker & Zigmond, 1990; Schumm et al., 1995).

Special educators also question the curricula and pedagogy advocated in the *Standards*. Newly proposed methods and materials are often at odds with the effective teaching model which was articulated by Good and his colleagues (Good & Grouws, 1979; Good, Grouws, & Ebmeier, 1983) and later embraced by mathematics researchers in special education (Darch, Carnine, & Gersten, 1984; Kelly, Gersten, & Carnine, 1990; Gleason, Carnine, & Boriero, 1990). Some special educators suggest that the instructional methods and materials proposed in the *Standards* are particularly ill-suited to the needs of academically low achieving students and those with learning disabilities because they are "too discovery-oriented" (e.g., Carnine, et al., 1994; Hofmeister, 1993). They also suggest that the *Standards* are nothing more than a recycling of old reforms (i.e., the New Math movement of the early 1960s). Finally, Hofmeister (1993) argues at length that the *Standards* are elitist, that what is generally proposed has little or no empirical validation.

Even those special educators who appear more sympathetic to the *Standards* exhibit difficulty and confusion when attempting to translate the mathematics research of the 1980s into a special education framework. Gersten, Keating, and Irvin (1995), for example, misconstrue constructivist discourse as teacher-directed example selection.

Also, traditional cognitive interpretations of student misconceptions in arithmetic are uncritically equated with constructivist theory.

Without systematic evaluation, the ways in which current mathematics reform might "play out" for students with learning disabilities or those at risk for special education is likely to remain speculative or only at the level of policy debate. At the very least, such evaluation would help determine whether any problems with innovations in mathematics rest in the nature of the curriculum and pedagogy or the more traditional problem of educating students with learning disabilities in mainstreamed environments.

Purpose of the Study

The purpose of this study was to examine the effects of an innovative approach to mathematics instruction on academic performance of mainstreamed students with learning disabilities and academically low achieving students who are at risk for special education. This research was part of an extensive study of teachers in three elementary schools, two of which were in the third year of using a new, university-based math reform curriculum. Nine third grade classrooms were the focus of systematic observations, teacher and student interviews, and academic assessment. Quantitative as well as qualitative data were collected in the attempt to triangulate on the effects of innovative curriculum and teaching techniques on target students (see Patton, 1980). Because of the extent of the data, this report will concentrate on the academic growth of students over the course of the year. Observation and interview data are described elsewhere (see Baxter & Woodward, 1995).

Method

Participants

Teachers and schools. The participants in this study were nine third grade teachers and their students in three schools located in the Pacific Northwest. The two intervention schools were selected because they were using the *Everyday Mathematics* program (Bell et al., 1993), which is closely aligned with the 1989 NCTM *Standards*. A third school, which acted as a comparison, was using *Heath Mathematics* (Rucker, 1988), a more traditional approach to mathematics. Five third grade teachers taught in the two intervention schools and four in the comparison school.

The schools were comparable along many variables. All were middle class, suburban elementary schools with similar socio-economic status (determined by the very low number of students on free or reduced lunch), as well as other demographic information provided by the districts.

Schools were also comparable in the general beliefs held by the staff regarding mathematics instruction. First through fifth grade teachers at each school completed the Mathematics Beliefs Scale (Fennema, Carpenter, & Loef, 1990), an updated version of the Teacher Belief Scale (Peterson, Fennema, Carpenter, & Loef, 1989). This measure has been used in a number of studies investigating the effects of innovative mathematics instruction. Differences between the staffs at the intervention and comparison schools were non-significant ($t_{(1,41)} = .94$; $p = .36$) on this scale.

Students. A total of 104 third grade students at the two intervention schools participated in this year long study. At the comparison school, 101 third graders participated. Forty-four students from the intervention and comparison schools were excluded from the data analysis because they were not present for either the pretesting or posttesting. Twelve students were classified as learning disabled on their IEPs, and they were receiving special education services for mathematics in mainstreamed settings. Seven students with learning disabilities were in the intervention schools and five were in the comparison school.

It should be noted that interviews with teachers in all three schools indicated that more students could have been referred for special education services in mathematics but were not for a variety of reasons. Some teachers mentioned that the special education teacher primarily served low incidence students (e.g., autistic, students with physical disabilities) or students who had reading problems. There was "little room left" to serve students for math.

Three teachers in the intervention schools chose not to refer students, and in two cases, they retained students in the general education classroom for mathematics instruction -- because they did not want to contend with the logistical problems of sending students out for mathematics at important or inconvenient times in the day. These teachers were also skeptical of the quality of mathematics instruction in the special education classroom. They felt that the traditional direct instruction approach to the subject did little to teach students the mathematics they needed for success in future grades.

Consequently, a wider pool of students was selected as a focus for this study. The mathematics subtest of the ITBS, administered in October, was used as a basis for further identifying students who were at-risk for special education services in mathematics. The 34th percentile was used as a criterion for selecting these students. In addition to the seven students with learning disabilities at two intervention schools, nine other students were identified based on total subtest performance on the ITBS. At the comparison school, another 17 students were identified. This resulted in a total of 16 students at the intervention schools and 22 at the comparison school who were considered academically low achieving in mathematics or were identified as having a learning disability in mathematics.

Materials

Intervention schools curriculum. As mentioned earlier, the two intervention schools in this study were using the *Everyday Mathematics* program. This program reflects over six years of development efforts by mathematics educators at the University of Chicago School Mathematics Project (UCSMP). The project has been funded by grants from the National Science Foundation as well as several major corporations. Initially, program developers translated mathematics textbooks from over 40 countries. Comparative analysis of elementary school texts indicated that the United States had one of the weakest mathematics curricula in the world (Usiskin, 1993). Among the many shortcomings, important mathematical concepts were taught too slowly, tasks surrounding concepts (e.g., measurement, geometry) were too simplistic, and there was too much repetition (Flanders, 1987).

To remedy these problems, developers at UCSMP created a curriculum that de-emphasized computations and changed the way concepts were reintroduced. For example, when major concepts reappear later in the year or in the next grade level, they are presented in greater depth. This structure is common to Japanese mathematics curricula (Stevenson & Stigler, 1992; Stigler & Baranes, 1988).

The UCSMP materials also emphasize innovative forms of problem solving. Unlike traditional math word problems, which are often conducive to a key word approach, problems or "number stories" are taken from the child's everyday world or from life science, geography, and other curriculum areas. The program developers are in strong agreement with other mathematics educators (e.g., Carpenter, 1985) in their view that students come to school with informal and intuitive problem solving abilities. The developers drew on this knowledge as a basis for math student-centered problem solving exercises. In these exercises, students are encouraged to use or develop a variety of number models which display relevant quantities (e.g., total and parts; start, change, end; quantity, quantity, difference) to be manipulated in solving these problems. While the third grade level of *Everyday Mathematics* is rich in problem solving, very few of the exercises consist of the one- and two-step problems that commonly appear in traditional commercial curricula for general and special education students.

Automaticity practice is achieved through the use of math "games." Students roll dice and add or subtract the numbers as a way of practicing math facts. Concepts are also developed through games. For example, two students alternate drawing cards from a deck and place each card in one of eight slots on a board. The goal of the game is to create the largest number eight-digit number. Developers suggest that this activity reinforces an understanding of place value in a game-like context.

The *Everyday Mathematics* program emphasizes a series of important NCTM *Standards*. Students spend considerable time identifying patterns, estimating, and developing number sense. The are encouraged to come up with multiple solutions for problems. Finally, the students are taught to use an array of math tools and manipulatives (e.g., calculators, scales, measuring devices, unifix cubes), and these materials play an important role in daily lessons.

Comparison school curriculum. The comparison school used the *Heath Mathematics Program.*, a traditional approach to mathematics. Lessons are structured around a systematic progression from facts to algorithms with separate sections on problem solving. Facts and algorithms are taught through massed practice, and students can be assigned as many as 50 facts and 20 to 30 computational problems at a time. Story problems involve one or two sentences and are generally of one type (i.e., they are directly related to the computational problems studied in the lesson or unit). Unlike the *Everyday Mathematics* program, there is far less emphasis on mathematical concepts and a much greater focus on computational problems. Teachers in the comparison school often supplemented the *Heath* program with worksheets containing more facts, computational problems, and occasional math exploration activities.

Procedures

Observational, interview, and academic performance data were collected over the 1993-94 school year. All third grade students in the three participating schools were administered the mathematics subtest of the Iowa Test of Basic Skills during the third week in September and again in the last week of April. In addition to this traditional measure of mathematics achievement, a stratified sample of third graders was given an innovative test of problem solving ability. ITBS problem solving subtest and total test scores were used as a basis for randomly selecting students in the intervention and comparison schools. ITBS scores were matched and t-tests were performed to determine comparability of the samples. This process continued until there were non-significant differences between the intervention and comparison groups ($t_{(1,38)} = .80$; $p = .38$ for problem solving; $t_{(1,38)} = .11$; $p = .75$ for total test score). The Informal Mathematics Assessment, which is described below, was administered to a total of 20 students in the two intervention schools and 20 comparable students in the comparison school during mid-October and again during the first week of May.

The nine participating teachers were systematically observed two to three times per week throughout the course of the year. Researchers interviewed the teachers informally during the year and formally in June at the end of school. Details of the observational instruments and findings as well as the interviews can be found elsewhere (see Baxter & Woodward, 1995).

Measures

Two different measures were administered to assess the effects of the intervention. The third grade level (Form G) of the Iowa Test of Basic Skills was used as both a pretest and as a posttest. The norm referenced test has well documented reliability and validity. It is a highly traditional, multiple choice form of assessment which measures computations, concepts, and problem solving skills.

The second measure, the Informal Mathematics Assessment (IMA), was an individually administered test of problem solving abilities. The intent of this measure was to examine the problem solving processes or strategies a student used in deriving an answer, as well as the answer itself. In this respect, it is consistent with the call for assessment which is more closely aligned with math reform and the NCTM *Standards* (Romberg, 1995). Students were also given a range of mathematical tools and representations which they were encouraged to use as part of the problem solving. The IMA "tool kit" included a calculator, ruler, paper and pencil, poker chips, and number squares with ones, tens, and hundreds values.

The six items on the test were based on an analysis of third grade mathematics texts, innovative materials which subscribe to the 1989 NCTM *Standards* as well as more traditional texts. In order to prevent fatigue and possible frustration, particularly with academically low achieving students, the items on the IMA were relatively brief, and the examiner read each one to the student. While the IMA took approximately 15 minutes to administer, students were given as much time as they wanted to complete each item. Alternate form reliability for the pre- and posttest versions of this measure was .87.

Figure 1 presents a word problem from the IMA. As with other word problems on the test, it was written to exclude key words (e.g., *each* and *every* often are taught as key words which signify multiplication or division). After the examiner read the problem to the student, s/he carefully noted if the student reread the problem, what calculations were made, and what tools or manipulatives were used. Finally, s/he asked the student to, "Tell me how you got that answer." This form of inquiry has been shown to be a valid method of determining how young children solve mathematics problems (Siegler, 1995). All sessions were tape recorded and transcribed for later scoring and qualitative analysis.

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insert Figure 1 about here

Individual student protocols were scored with a rubric which was analytically derived from the NCTM *Standards* and related literature on innovative mathematics assessment (Lesh & Lamon, 1992). A five point scale was used for each item, with the highest score reflecting both the quality of the student's answer as well as the process used to derive the answer. Inter-rater reliability for scoring the student protocols was .93.

Finally, the IMA protocols were subjected to a categorical analysis. Researchers examined student answers in an effort to classify different kinds of problem solving behavior. The extent to which students used manipulatives provided in the tool kit (particularly paper, pencil, and calculators) and the strategies they used to solve problems (e.g., guessing, using numbers provided in the problem in random order, decomposing problems into subunits) were analyzed. Inter-rater reliability for the categorical analysis was .88.

Results

Data for this study were analyzed quantitatively and qualitatively. The quantitative data provided a broad framework for gauging the relative changes in academic performance for students at the intervention and comparison schools. This was particularly important as two different types of academic measures were used to assess growth in mathematics. The protocols from the IMA, along with classroom observations and teacher interviews enabled a qualitative analysis of the effects of the innovative curriculum on students with learning disabilities and academically low achieving students.

The ITBS

The ITBS functioned as a traditional measure of achievement. Pretest scores from the fall for the total test and all subtests were used as covariates in an Analysis of Covariance (ANCOVA). Results are presented for the total sample and the three ability groups.

Total sample. Results of the ANCOVA show a significant difference between groups ($F_{(1,202)} = 29.12$, $p < .001$) on the concepts subtest, favoring the intervention group. All other differences were statistically non-significant. Table 1 provides descriptive statistics for the two groups on the total test and all three subtests. Generally, students at the two intervention schools indicated mixed growth over the year as measured by the ITBS. Mean percentiles for fall and spring indicate that total test performance was stable, with noticeable increases in the area of concepts and slight to considerable decreases in computations and problem solving, respectively. The comparison students declined slightly over the course of the year in all areas.

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insert Table 1 about here

Analysis by ability group. ANCOVAs were performed in a similar manner for students at the three different ability levels as determined by the total test score on the ITBS in the fall. Academically low achieving students, which included the 12 mainstreamed students with learning disabilities in mathematics, scored at or below the 34th percentile. Average ability students scored from the 35th to the 67th percentile, and high ability students scored above the 67th percentile.

Results of the ANCOVAs for the academically low achieving students indicated non-significant differences for the total test and all three subtests. Table 2 provides descriptive statistics for these two groups of students who scored below the 34th percentile in the fall on these measures. In general, students in both schools showed modest improvement. The most dramatic gains were in problem solving for the intervention students and in total score for the comparison students.

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insert Table 2 about here

ANCOVA results for average ability students were significant only in the area of concepts. Like the total sample comparisons, the results favored the intervention students ($F_{(1,66)} = 8.05$, $p < .01$). All other differences were non-significant.

For the high ability students, ANCOVA results indicated significant differences favoring the intervention students on concepts ($F_{(1,95)} = 12.75$, $p < .001$) and problem solving ($F_{(1,95)} = 5.12$, $p = .03$). Descriptive statistics for average and high ability students for the intervention and comparison groups on these measures are provided in Table 3.

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Informal Assessment of Mathematics (IMA)

An ANCOVA was performed on spring test results of the IMA for the total sample of students tested (i.e., 20 per condition). The fall IMA test scores were used as a covariate. Results strongly favor students in the intervention group ($F_{(1,37)} = 9.85$, $p < .01$).

Data were further analyzed by ability group. Due to the small sample sizes, further ANCOVAs were not conducted for high, average, and low ability groups. Instead, those data are presented descriptively in Table 4 along with the descriptive data for the total sample. Data for the three ability groups are also presented graphically in Figure 2. Data suggest that the greatest effects, at least by ability, were for the average students (i.e., those between the 34th and 67th percentile).

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insert Table 4 and Figure 2 about here

Qualitative analysis of IMA protocols. The primary purpose of this year-long case study was to investigate the effects of an innovative curriculum like *Everyday Mathematics* on students with learning disabilities and those at-risk for special education. Therefore, protocols of all of the students in the intervention school who were given the IMA were carefully analyzed along a variety of dimensions.

Protocols were first examined categorically using constructs associated with the scoring rubric as well as the theoretical guidelines used to develop the IMA (e.g., those emanating from the 1989 NCTM *Standards*; recent research, particularly on innovative assessment in mathematics). Transcribed protocols and examiner notes taken during the individualized administration of the IMA enabled researchers to determine the extent to which students used manipulatives, calculators, paper and pencil, and the "reasoning" used to derive answers to specific problems.

Categorical analysis of protocols by ability groups across time indicated some similar behavior among all of the students. There were no discernible differences, for example, in the use of manipulatives as part of the problem solving process. By spring, all students tended to increase their use of paper and pencil for problem solving. The extent to which students in different ability groups used calculators remained constant, with high ability students using calculators over twice as frequently as academically low achieving students (71% versus 29%).

The most noticeable differences between students of different academic abilities were evident in the way students reasoned out problems, particularly the longer, more complex word problems shown in Figure 1. Three distinct categories of student reasoning emerged from the protocols which can be related to the problem solving literature in mathematics. These categories are shown in Figure 3 below.

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insert Figure 3 about here

The first category of Confusion and Uncertainty most directly pertains to students with learning disabilities and other academically low achieving students. As the data indicate, these students continued to guess, merely repeat numbers presented in the problem, or quickly respond, "I don't know," once the examiner finished reading the problem. Even with prompts or gentle attempts to get them to work a part of the problem, the students often appear to have little or no framework for simplifying a problem. Average ability students are far less likely to react this way by spring.

If there was any shift in this categorical behavior among the academically low achieving students, it was to move from giving up on the problem in the fall to an attempt to use numbers in the problem, albeit incorrectly in the spring. Figure 4 below presents a protocol for Problem 6 in the fall and its alternate version in the spring. The spring version of the problem, not shown in Figure 1, describes the collection of box tops for playground equipment. Like the fall version of the problem, students are given extraneous information and if they answer it correctly, they generally do so in three steps. The correct answer is 7618 box tops.

Figure 4 is a protocol of a mainstreamed student with learning disabilities in one of the intervention schools. The shift in the way he works the problem reflects a common pattern found among the lowest third of students: numbers presented in the problem are used, but with no association to the correct operations or categories. To solve the problem correctly, the student would need to 1) multiply the 19 box tops times the 97 third graders 2) multiply 35 box tops times the 165 fifth graders and 3) add the two products together. As the protocol indicates, the student with learning disabilities uses relevant as well as irrelevant information (e.g., 28 box tops for fourth graders) in the linear order presented in the problem.

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insert Figure 4 about here

In contrast to the academically low achieving students, average and high ability students spent more time conceptualizing the IMA problems before they worked them. For example, when working Problem 5 presented in Figure 1, many students used an "if-then" logic to talk through the problem prior to computing it on paper or using a calculator. This verbal restatement served as an important way to mediate what would have otherwise been an immediate and incorrect answer (usually in the form of adding or multiplying the distance from home to school twice, ignoring the intermediate 238 steps of walking back home to get the book). Moreover, Problem 6 was a clear occasion for high ability students (and many average students by spring) to carefully discern the relevant information from the problem and divide it into subproblems. Again, students restated the problem verbally in a simplified form as they worked it on paper or used a calculator. Both the conditional logic and the tendency to clearly decompose a problem into relevant subproblems was missing with the academically low achieving students.

Discussion

The results of this study suggest that the innovative curriculum benefited the majority of students in the intervention schools. Quasi-experimental comparisons indicated no overall decline in ITBS total test scores for the entire sample. In fact, most intervention students maintained or significantly improved performance levels on ITBS subtests directly related to the design of the intervention curricula (i.e., concepts and problem solving for average and higher ability students).

Improved performance was also evident on the IMA alternative assessment, a measure which is more closely aligned with recent reforms in mathematics. Quantitative and qualitative changes on the IMA were particularly evident for average achieving students at the intervention schools. They tended to more closely approximate the behavior of high achieving students in their ability to restate and decompose problems as well as use calculators as an integral part of problem solving.

Some mathematics reformers (e.g., Romberg, 1995) may view these findings as highly encouraging insofar as performance at the intervention school was not undercut by a lowering of scores on traditional measures. The findings from the IMA in this study tend to complement overall trends in the ITBS data. As Romberg and other would argue, an innovative form of assessment like the IMA is critical in documenting the varied and more subtle effects of mathematics reform.

As for students with learning disabilities and their academically low achieving peers, data from this study indicate only marginal improvement in their learning. Quasi-experimental results even suggest that students at or below the 34th percentile in the comparison school made more dramatic gains in total test performance on the ITBS total test (i.e., from the 20th to 30th percentile versus 24th to 26th percentile) and ITBS Computations subtest than similar students at the intervention schools. Surprisingly, low achieving students in both intervention and comparison schools made impressive gains on the problem solving subtest of the ITBS, at least in terms of percentile change.

Changes on the IMA for these students were much more modest, particularly for students in the comparison school where their mean performance over time remained at the same 40 percent correct level. Low ability students at the intervention school fared better, but their gains were not comparable to average ability students. Spring scores were still below 50 percent correct on this measure. Moreover, qualitative analyses of the data indicate that these students still exhibited high levels of confusion and uncertainty when answering many of the IMA problems, and tended to just repeat numbers rather than conceptualize and logically simplify complex problems. Unlike their average and above average peers, they struggled to incorporate calculators into their problem solving process.

Although some might interpret these data as supporting special educators' criticisms of the current mathematics reform, we hesitate to do so. In fact, the general success of students at the intervention schools raises a series of complex questions which go well beyond the polemics against the *Standards* in the recent special education literature.

An evaluation of the direct impact of the 1989 NCTM *Standards* would be a difficult, if not an impossible endeavor. Few in the mathematics education community would suggest that the *Standards*, which were designed as a framework for reform, provide a sufficient blueprint for daily instruction. For this reason, current study investigated an innovative curriculum, one which was closely aligned with the *Standards* but based on other sources (e.g., the translation of elementary and secondary textbooks from other countries which consistently score favorably in international comparisons). As a university, research-based effort, the curriculum also reflects field testing in a variety of settings and multiple revisions. Essentially, the *Everyday Mathematics* program represents the *Standards* and much more.

Viewed in this light, data from this study do not support the contention of critics from special education that reform efforts which represent the *Standards* are elitist. Rather, the data clearly suggest that the curriculum benefited the majority of students. Observations and interviews conducted as part of this study (Baxter & Woodward, 1995) indicated that a teacher's capacity to meet the needs of the lowest achieving students was complicated by many factors, only part of which may have been due to the structure and content of the curriculum. Equally problematic were the limited educational resources available to mainstream teachers (e.g., personnel, contact time, specific pedagogical techniques). If anything, these findings are consistent with recent mainstreaming research, which suggests that a variety of classroom organizational, instructional, and institutional variables inhibit the success of these students when they are taught in regular education settings (e.g., Baker & Zigmond, 1990; Schumm et al., 1995).

Therefore, the fact that the innovative curriculum met the needs of the majority of students in the intervention schools is cause for special educators to begin reconsidering the adequacy of many of their current instructional practices. It should be remembered that with the innovative curriculum, students discuss multiple solutions to problems, defend problem solving methods, and use an array of tools to work out solutions and demonstrate answers These practices differ substantially from current special education and past general education methods.

A careful analysis of even the most widely cited special education methods for teaching mathematics suggests a considerable difference in structure and content (Woodward, Baxter, & Scheel, in press). Special education curricula tend to place excessive emphasis on acquisition of facts, a rote mastery of the algorithms for basic operations, and key word solutions to traditional one- and two-step word problems (see Darch, et al., 1984; Silbert, Carnine, & Stein, 1989). Cognitive-based research not only questions whether teaching algorithms (VanLehn, 1990; Woodward & Howard, 1994) or problem solving (Hegarty & Mayer, 1993) can be successful over the long term, but more significantly, if these kinds of instructional experiences adequately prepare students for the kind of learning found in the intervention classrooms in this study. As Resnick (1989) has suggested, new forms of literacy do not follow a traditional hierarchy of preskills to a final point where students actually solve complex, ill-defined problems. Instead, skills need to be mixed with challenging activities.

It would appear, then, that a continued focus on (and condemnation of) the 1989 NCTM *Standards* is misplaced. Given the direction of research in mathematics education over the last two decades, the profound changes in technology which have devalued rote computational abilities, and findings such as the ones in this study, more attention should be placed on new instructional approaches for students with learning disabilities.

Implications for Practice

A lengthy discussion of instructional strategies which would address the needs of students with learning disabilities and those at risk for special education in innovative mathematics classrooms would go well beyond the intent of this article. In the time following the research reported above, however, action and empirical, intervention studies have been conducted in the attempt to craft strategies for academically low achieving students (Baxter, Woodward, Olson, & Kline, 1996; Woodward & Baxter, 1996). Efforts to date suggest two levels of intervention.

First, new forms of literacy in mainstreamed settings, ones which promote classroom discourse and small group activities, argue for significant changes in classroom organization. Slavin, Madden, Karweit, Livermon, and Donlan's (1990) work in deploying students for small group, homogeneous instruction during portions of a lesson holds promise for innovative mathematics classrooms. This practice generally requires a cooperative working relationship for grade level teachers and additional instructional assistance. This latter role may be fulfilled by special educators or paraprofessionals working in mainstreamed environments. Small, homogenous group instruction affords opportunities to adjust content to the ability level of students and to increase student discourse and teacher feedback. Deployment is based on the content of the week's lesson and the particular needs of the lowest achieving students. In some instances where the students are learning a new topic (e.g., geometry), they will remain in larger, heterogeneous groups. This dynamic or contingent grouping is designed to overcome traditionally rigid patterns of ability grouping which tend to persist throughout an entire lesson.

Specific pedagogical techniques comprise a second level of intervention. Work over the last decade in reading (Bos & Anders, 1990; Palinscar & Klenk, 1992) and writing (Englert, Raphael, & Anderson, 1992) provide important insights into the ways in which complex forms of literacy can be modified for students with learning disabilities through a balance of explicit strategies, a careful attention to cognitive process (e.g., the methods a student uses to derive an answer, the quality of a student's explanation) over product, and teacher-student dialogue. Again, deployment creates a context for tailoring these techniques to students who are experiencing the greatest difficulties. Yet it should be noted that techniques such as scaffolding or strategic feedback need to be understood in a *content dependent* fashion. Broad instructional principles such as those commonly associated with the effective teaching literature are likely to be insufficient. Instead, advances in our understanding of how students with learning disabilities might benefit from new approaches to mathematics fully depend upon innovative curricula and a teacher's subject matter knowledge. This position is consistent with much of the professional development work in contemporary mathematics education.

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